Math 344 Winter 2026 Exam Two

Name:________________________________________

- Put your name in the "________________________________________" above.
- Answer all questions.
- Proofs are graded for clarity, rigor, neatness, and style.
- You may choose **two** of the three proofs for me to grade; write your choices below:

  Proof one: __________ Proof two: __________.

  (If you leave this blank, I will grade the first two proofs.)
- Good luck!

## Computations

1. (a) Write down three subgroups of $S_6$ that have size 3.
   (b) Write down three elements of $S_6$ of order 4.
   (c) Write down a group of size 32 with no elements of order 16.

2. Let $G = \mathbb{Z}_{20}$ and let $H$ be the subgroup of $G$ generated by 5.

   (a) Write down all elements of all right cosets of $H$.

   (b) What is $(G : H)$?

# Proofs

(I) Suppose that $G, H$ are groups with identities $e_g, e_H$. Now assume that $G \times H$ is cyclic, so there are some $g_0 \in G$ and $h_0 \in H$ such that $G \times H = \langle (g_0, h_0) \rangle$. Prove that $G = \langle g_0 \rangle$.

(II) Suppose $G, H$ are groups and $\phi\colon G \to H$ is an isomorphism. Prove: if $H$ is cyclic, then $G$ is cyclic.

(III) Let $G$ be a group and define a relation $\sim$ on $G$ by: for all $g, h \in G$

$$g \sim h \qquad \text{if and only if} \qquad \text{there exists } j \in \mathbb{Z} \text{ such that } g^j = h^j.$$

Prove that $\sim$ is an equivalence relation.

## Extra Credit (if you have extra time)

Suppose that $G$ is an abelian group of size 64. Define the function

$$\begin{aligned} \phi\colon G &\to G \\ g &\mapsto g^2 \end{aligned}$$

Either prove that $\phi$ is an isomorphism or prove it is not an isomorphism.