Math 344 Fall 2025 Exam Two

Name:______________________________________________

- Put your name in the "______________________________________________" above.
- Answer all questions.
- Proofs are graded for clarity, rigor, neatness, and style.
- Good luck!

## Computations

1. For the following element $f$ of $S_9$, do the following:
   (a) write $f$ in disjoint cycle form,
   (b) write $f$ as a product of transpositions,
   (c) state the parity of $f$, and
   (d) write $(126) \circ (389) \circ f$ in disjoint cycle form.

$$
\begin{gathered}
f\colon \{1,2,3,4,5,6,7,8,9\} \to \{1,2,3,4,5,6,7,8,9\} \\
1 \mapsto 7 \\
2 \mapsto 2 \\
3 \mapsto 4 \\
4 \mapsto 5 \\
5 \mapsto 3 \\
6 \mapsto 1 \\
7 \mapsto 8 \\
8 \mapsto 9 \\
9 \mapsto 6
\end{gathered}
$$

2. Let $H$ be the subgroup of $S_5$ generated by $(12345)$. In other words, let $H = \langle(12345)\rangle$. Write, in disjoint cycle form,

   (a) all elements of $H(12)$, and

   (b) all elements of $(12)H$.

## Proofs

(I) Suppose $G, H$ are groups and $\phi: G \to H$ is an isomorphism. Prove: if $G$ is cyclic, then $H$ is cyclic.

(II) Suppose that $p$ is a prime number and that $G$ is a group of size $p$, say with identity $e$. Prove: for all $g \in G$, if $g \neq e$, then $G = \langle g \rangle$.

(III) Suppose that $G, H$ are groups and $\phi\colon G \to H$ is an isomorphism. In particular, we know that $\phi$ is bijective, so it has an inverse: $\phi^{-1}\colon H \to G$. Prove that $\phi^{-1}$ is an isomorphism.

## Extra Credit (if you have extra time)

Suppose that $G$ is a abelian group (ie, the operation of $G$ is commutative) of size 77. Define

$$\begin{aligned} \phi\colon G &\to G \\ g &\mapsto g^3. \end{aligned}$$

Prove that $\phi$ is an isomorphism.