Math 344 Fall 2025 Exam One

Name:____

- Put your name in the "____" above.
- Answer all questions.
- Proofs are graded for clarity, rigor, neatness, and style.
- Good luck!

Computations

1. For the following element f of S_9, do the following:
   (a) compute $f \circ f \circ f \circ f(9)$,
   (b) write the smallest positive integer n such that
   $$\overbrace{f \circ \cdots \circ f}^{n \text{ times}}(9) = 9,$$
   and
   (c) write down an element m in $\{1,2,3,4,5,6,7,8,9\}$ with the property that $f \circ f \circ f(m) = m$.

$$\begin{aligned} f\colon \{1,2,3,4,5,6,7,8,9\} &\to \{1,2,3,4,5,6,7,8,9\} \\ 1 &\mapsto 7 \\ 2 &\mapsto 2 \\ 3 &\mapsto 4 \\ 4 &\mapsto 5 \\ 5 &\mapsto 3 \\ 6 &\mapsto 1 \\ 7 &\mapsto 8 \\ 8 &\mapsto 9 \\ 9 &\mapsto 6 \end{aligned}$$

2. Suppose that G is a group with a subgroup H. For any $g \in G$, define $gH = \{gh \mid h \in H\}$.

   (a) Write down two elements A of D_4 such that $F \in A\langle R \rangle$. (For example, we see that $A = I$ does not work since $I\langle R \rangle = \{I, R, R^2, R^3\}$ does not contain F.)

   (b) Write down three distinct subgroups of D_4 of size 2.

   (c) Write down three elements B in D_5 such that $\langle B \rangle = \langle R \rangle$.

Proofs

(I) Let $H = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} \mid \text{ both } m \text{ and } n \text{ are even}\}$. Prove that H is a subgroup of $\mathbb{Z} \times \mathbb{Z}$.

(II) Suppose that G is a group and write

$$H = \left\{ g^{130} \mid g \in G \right\}.$$

Prove: if G is commutative, then H is a subgroup of G.

(III) Define the operation $\star$ on $\mathbb{R}$ by setting $x \star y = x^2 + y^2$. Either prove that $(\mathbb{R}, \star)$ is a group or prove that it is not a group.

Extra Credit (if you have extra time)

Suppose that G is a group with two subgroups I and J. Prove that

$$I \cup J \text{ is a subgroup of } G \qquad \text{if and only if} \qquad \text{either } I \subseteq J \text{ or } J \subseteq I.$$