Math 344 Fall 2025 Final Exam

Name:____

- Put your name in the "____" above.
- Answer all questions.
- Proofs are graded for clarity, rigor, neatness, and style.
- Good luck!

Computations

1. (a) Write down all elements of order 2 in D_6.

   (b) Write down all elements of order 3 in D_6.

2. Write down

   (a) An infinite group that is not cyclic,
   (b) An infinite noncommutative group,
   (c) A group of size 81 where every element has either order 3 or order 1, and
   (d) A noncommutative group of size 88.

3. Consider the group S_5, a group of size 120.

(a) Write down any $f_1 \in S_5$ such that $|S_5/\langle f_1\rangle| = 24$.

(b) Write down any $f_2 \in S_5$ such that $|S_5/\langle f_2\rangle| = 30$.

(c) Write down any $f_3 \in S_5$ such that $|S_5/\langle f_3\rangle| = 40$.

(d) Write down any $f_4 \in S_5$ such that $|S_5/\langle f_4\rangle| = 20$.

Proofs

(I) Suppose that G is a group with 77 elements, that H is a group, and that $\phi: G \to H$ is a homomorphism. Prove that if $\ker(\phi)$ contains fewer than 7 elements, then ϕ is injective.

(II) Suppose that G is a cyclic group and H is a normal subgroup of G. Prove that G/H is cyclic.

(III) Suppose that G_1, G_2 are groups, and define

$$\begin{aligned} \phi\colon G_1 \times G_2 &\to G_2 \\ (g_1, g_2) &\mapsto g_2. \end{aligned}$$

In HW7, we proved

- ϕ is a homomorphism and
- $\ker(\phi)$ is isomorphic to G_1.

Prove that $(G_1 \times G_2) / \ker(\phi)$ is isomorphic to G_2.