HW 6 Due: 21 May 2024

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

Computations

1. Let $G = \mathbb{Z}_{20}$ and $H = \langle 15 \rangle$. Write down all elements of all (right) cosets of H in G.
2. Let $G = D_4$ and $H = \langle R \rangle$. Write down all elements of all (right) cosets of H in G.
3. Define an equivalence relation on D_5 by setting, for all $g, h \in D_5$:

$$g \sim h \qquad \text{if and only if} \qquad \langle R \rangle\, g = \langle R \rangle\, h.$$

   For $g \in D_5$, write $[g]$ for the equivalence class of g using this relation. Write down all elements in $[F]$.

Proofs

(I) Let G be a group, let H be a subgroup of G, and let $a, b \in G$. Prove

$$Ha = Hb \qquad \text{if and only if} \qquad ab^{-1} \in H.$$

(II) Suppose n is a positive odd integer, that G is a group of order $2n$, and that $a, b \in G$ have orders $2, n$, respectively. Prove that

$$G = \left\{ a^i b^j \mid i \in \{0, 1\} \text{ and } j \in \{0, \dots, n-1\} \right\}.$$

(III) Let $\mathcal{F}$ be the set of all functions with domain and codomain $\mathbb{R}$. Define $\sim$ on $\mathcal{F}$ by setting for all $f, g \in \mathcal{F}$:

$$f \sim g \qquad \text{if and only if} \qquad f(0) = g(0).$$

Prove that $\sim$ is an equivalence relation on $\mathcal{F}$.