HW 4 Not for turning in

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work.*

1 Computations

1. Consider the function $g \in S_4$ given by

$$\begin{aligned} 1 &\mapsto 2 \\ 2 &\mapsto 1 \\ 3 &\mapsto 3 \\ 4 &\mapsto 4. \end{aligned}$$

   Write down all functions $f \in S_4$ with the property that

   (a) $f \circ f \circ f = \mathrm{id}_{\{1,2,3,4\}}$,

   (b) $f \circ g = g \circ f$,

   (c) $f \circ g = \mathrm{id}_{\{1,2,3,4\}}$.

   (d) $f(1) \in \{1,2\}$ and $f(2) \in \{1,2\}$.

2. How many elements (f,g) in $S_3 \times S_3$ are there that satisfy $(f,g)(f,g)(f,g) = \big(\mathrm{id}_{\{1,2,3\}}, \mathrm{id}_{\{1,2,3\}}\big)$?

2 Proofs

(I) Let A be a set and B any *finite* subset of A. Define

$$G = \{f \in S_A \mid \text{for all } b \in B,\ f(b) \in B\}.$$

Prove that G is a subgroup of S_A.

(II) Define the set

$$G = \{f \in S_5 \mid f(2) = 3\}.$$

Is G a subgroup of S_5? Prove your answer is correct.

(III) Let G be a group, and define

$$D = \{(g,g) \mid g \in G\}.$$

Assume that D is a subgroup of $G \times G$.[1] Prove that G is isomorphic to D.

[1] This is always true and in principle, we could prove it. However, I'm not requiring a proof. At this time.