HW 5 Due: 8 May 2024

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

Computations

1. Write down all the orders of all the elements of D_4.
2. Write down all the orders of all the elements of $S_3 \times \mathbb{Z}_2$.
3. Let $f = (14563) \in S_9$. Write down all the orders of all the elements of $\langle f \rangle$.

Proofs

(I) Let G be a group and suppose $g, h \in G$ have finite order. Prove: if $gh = hg$, then $\operatorname{ord}(gh)$ is a divisor of $\operatorname{lcm}(\operatorname{ord}(g), \operatorname{ord}(h))$.

(II) Let G be a group, and define

$$D = \{(g, g) \mid g \in G\}.$$

Assume that D is a subgroup of $G \times G$.[1] Prove that G is isomorphic to D.

(III) Suppose that G, H are groups and $\phi: G \to H$ is an isomorphism. Prove: for all $g \in G$,

$$\operatorname{ord}(g) = \operatorname{ord}(\phi(g)).$$

References

[Pin10] Charles C. Pinter, *A book of abstract algebra*, Dover Publications, Inc., Mineola, NY, 2010, Reprint of the second (1990) edition [of MR0644983]. MR 2850284

[1] This is always true and in principle, we could prove it. However, I'm not requiring a proof. At this time.