HW 4 Due: 1 May 2024

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

1 Computations

1. Consider the function $g \in S_4$ given by

$$1 \mapsto 2$$
$$2 \mapsto 1$$
$$3 \mapsto 3$$
$$4 \mapsto 4.$$

   Write down all functions $f \in S_4$ with the property that

   (a) $f \circ f \circ f = \mathrm{id}_A$,
   (b) $f \circ g = g \circ f$,
   (c) $f \circ g = \mathrm{id}_{\{1,2,3,4\}}$.
   (d) $f(1) \in \{1,2\}$ and $f(2) \in \{1,2\}$.

2. How many elements (f,g) in $S_3 \times S_3$ are there that satisfy $(f,g)(f,g)(f,g) = \left(\mathrm{id}_{\{1,2,3\}}, \mathrm{id}_{\{1,2,3\}}\right)$?

2 Proofs

(I) Let A be a set and B any *finite* subset of A. Define

$$G = \{f \in S_A \mid \text{for all } b \in B, \ f(b) \in B\}.$$

Prove that G is a subgroup of S_A.

(II) Define the set
$$G = \{f \in S_5 \mid f(2) = 3\}.$$

Is G a subgroup of S_5? Prove your answer is correct.