HW 2 Due: 22 April 2024

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work.*

1 Computations

1. Write down every element of $\mathbb{Z}_3 \times \mathbb{Z}_3$, and write down its inverse. (For one example, note that the element $(0,0)$ has inverse $-(0,0) = (0,0)$.)
2. Write down every multiple of $(1,1)$ in the group $\mathbb{Z}_6 \times \mathbb{Z}_3$.
3. Write down three elements (a,b) of $\mathbb{Z}_6 \times \mathbb{Z}_3$ with the property
$$|\{n(a,b) \mid n \in \mathbb{Z}\}| = 3.$$

2 Proofs

(I) Let G be a group with identity elements e_1, e_2. Prove that $e_1 = e_2$.

(II) Let G, H be groups. Prove that if G, H are both abelian, then $G \times H$ is abelian.

(III) Let G be a group, and let $g, h \in G$. Assume that
$$\text{for all } x \in G, \text{ we have } xg = gx.$$
Prove that
$$\text{for all } x \in G, \text{ we have } x\left(hgh^{-1}\right) = \left(hgh^{-1}\right)x.$$