Math 344 | Fall 2023 | Midterm

Name:____

- Put your name in the "____" above.
- Answer all questions.
- Proofs are graded for clarity, rigor, neatness, and style.
- Good luck!

Computations

1. For the following element f of S_9, do the following:

   (i) write f in disjoint cycle form,

   (ii) write f as a product of transpositions,

   (iii) state the parity of f, and

   (iv) write $(126) \circ (389) \circ f$ in disjoint cycle form.

$$
\begin{aligned}
f\colon \{1,2,3,4,5,6,7,8,9\} &\to \{1,2,3,4,5,6,7,8,9\} \\
1 &\mapsto 7 \\
2 &\mapsto 2 \\
3 &\mapsto 4 \\
4 &\mapsto 5 \\
5 &\mapsto 3 \\
6 &\mapsto 1 \\
7 &\mapsto 8 \\
8 &\mapsto 9 \\
9 &\mapsto 6
\end{aligned}
$$

2. Suppose that G is a group with a subgroup H. For any $g \in G$, define $gH = \{gh \mid h \in H\}$. In disjoint cycle form, enumerate all elements of gH in the following situations:

   (a) $G = S_3$, $H = \langle(123)\rangle$, $g = (132)$,

   (b) $G = S_3$, $H = \langle(123)\rangle$, $g = (12)$, and

   (c) $G = S_3$, $H = \langle(123)\rangle$, $g = (23)$.

Proofs

(I) Suppose that G is a group and let $D = \{(g, g) \mid g \in G\}$.

(a) Prove that D is a subgroup of $G \times G$.

(b) Prove that G is isomorphic to D.

(II) Suppose that G, H are groups and that $\phi: G \to H$ is an isomorphism. Prove: if G is commutative, then H is commutative.

(III) Suppose that G is a group with identity e, and choose some $g \in G$. Define the function

$$\begin{aligned} f_g \colon G &\to G \\ x &\mapsto gx. \end{aligned}$$

(a) Prove: f_g is bijective.

(b) Prove: if f_g is an isomorphism, then $g = e$.

Extra Credit (if you have extra time)

Suppose that G is a group with two subgroups I and J. Prove that

$$I \cup J \text{ is a subgroup of } G \qquad \text{if and only if} \qquad \text{either } I \subseteq J \text{ or } J \subseteq I.$$