Math 344   Spring 2020   Final

Name:_____

- Put your name in the "_____" above.

- The only resources you should be using are

    - your book [Pin10],
    - your notes,
    - videos of lecture, and
    - talking to me (Derek) in class on Monday.

- You can submit either

    - typed or written pdf documents or
    - text typed into d2l.

- Answer all problems.

- Good luck!

Proof questions

1. Suppose that G is a group of size 77. Suppose that H is a nontrivial normal subgroup of G. (In other words, suppose that $|H| > 1$.) Prove that G/H is cyclic.

2. Let $\phi\colon\mathbb{Z}_{100} \to \mathbb{Z}$ by any homomorphism of groups.

   (a) Prove that $\phi(1) = 0$.

   (b) What is $\ker\phi$? Prove your answer is correct.

3. Suppose that G is a group and $|G| < 200$. Suppose that G has subgroups of size 15 and 36. What is the size of G? Prove your answer is correct.

Computational questions

1. Using cycle notation, write down all elements of S_4 with order 2.

2. Using cycle notation, write down all elements of S_4 with order 4.

3. (a) Using cycle notation, write down one element of S_4 with order 3, and let's call it f. (And let's agree to write "id" for the identity element of S_4.)

(b) Using cycle notation, write down all elements of $\langle f \rangle$.

(c) What is $|S_4/\langle f \rangle|$?

(d) Write down all elements of $S_4/\langle f \rangle$.

References

[Pin10] Charles C. Pinter, *A book of abstract algebra*, Dover Publications, Inc., Mineola, NY, 2010, Reprint of the second (1990) edition [of MR0644983]. MR 2850284