

Exam One

Math 344

- Write your name here: _____.
- Answer all questions.
- Proofs are graded for clarity, rigor, neatness, and style.
- Good luck!

Computations

1. Consider the following two functions, to which we will give the names g and h for this problem:

$$\begin{array}{ll} g: \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \rightarrow \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & h: \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \rightarrow \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ \begin{array}{l} 1 \mapsto 3 \\ 2 \mapsto 2 \\ 3 \mapsto 4 \\ 4 \mapsto 6 \\ 5 \mapsto 7 \\ 6 \mapsto 1 \\ 7 \mapsto 9 \\ 8 \mapsto 1 \\ 9 \mapsto 5 \end{array} & \begin{array}{l} 1 \mapsto 3 \\ 2 \mapsto 6 \\ 3 \mapsto 3 \\ 4 \mapsto 3 \\ 5 \mapsto 8 \\ 6 \mapsto 3 \\ 7 \mapsto 5 \\ 8 \mapsto 1 \\ 9 \mapsto 9. \end{array} \end{array}$$

- (a) Compute $g \circ h(1)$,
(b) compute $h \circ g(1)$,
(c) write the smallest positive integer n such that

$$\overbrace{g \circ \cdots \circ g}^{n \text{ times}}(1) = 1,$$

and

- (d) write down an element m in $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ with the property that $h \circ h(m) = h(m)$.

2. (a) Write down three subgroups of D_6 that have size 2.
- (b) Write down all elements of $\mathbb{Z}_4 \times \mathbb{Z}_4$ that are their own inverse.
- (c) Write down three subgroups of $\mathbb{Z}_6 \times \mathbb{Z}_6$ of size 3.
- (d) Write down a group that has at least one subgroup of size 2 and at least one subgroup of size 7.

Proofs

1. Let $H = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} \mid 2m + 3n = 0\}$. Prove that H is a subgroup of $\mathbb{Z} \times \mathbb{Z}$.

2. Suppose that G is a group with a subgroup H . Suppose that $g_1, g_2 \in G$ satisfy $g_1 g_2 \in H$. Prove:
- if $g_1 \in H$, then $g_2 \in H$.

3. Define the operation $\star$ on $\mathbb{Z}_{\geq 3}$ by

$$m \star n = \begin{cases} m & \text{if } m \geq n \\ n & \text{if } n \geq m. \end{cases}$$

- (a) Prove that $\star$ admits an identity element.
- (b) Either prove that $(\mathbb{Z}_{\geq 3}, \star)$ is a group or prove that it is not a group.