

# HW4

MTH 344

## Instructions

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

## 1 Computations

1. Consider the function  $g \in S_4$  given by

$$\begin{aligned}1 &\mapsto 2 \\2 &\mapsto 1 \\3 &\mapsto 3 \\4 &\mapsto 4.\end{aligned}$$

Using cycle notation, write down all functions  $f \in S_4$  with the property that

- (a)  $f \circ f \circ f = \text{id}_{\{1,2,3,4\}}$ ,
  - (b)  $f \circ g = g \circ f$ ,
  - (c)  $f \circ g = \text{id}_{\{1,2,3,4\}}$ .
  - (d)  $f(1) \in \{1, 2\}$  and  $f(2) \in \{1, 2\}$ .
2. How many elements  $(f, g)$  in  $S_3 \times S_3$  are there that satisfy  $(f, g)(f, g)(f, g) = (\text{id}_{\{1,2,3\}}, \text{id}_{\{1,2,3\}})$ ?

## 2 Proofs

1. Let  $A$  be a set and  $B$  any *finite* subset of  $A$ . Define

$$G = \{f \in S_A \mid \text{for all } b \in B, f(b) \in B\}.$$

Prove that  $G$  is a subgroup of  $S_A$ .

2. Define the set

$$G = \{f \in S_5 \mid f(2) = 3\}.$$

Is  $G$  a subgroup of  $S_5$ ? Prove your answer is correct.

3. This is a trickier one! Suppose that  $n$  is a positive, and suppose that  $\alpha$  is a nontrivial cycle in  $S_n$ . That is, there is a  $j \in \{2, \dots, n\}$  and distinct integers  $a_1, \dots, a_j$  such that

$$\alpha = (a_1 \ a_2 \ \cdots \ a_j).$$

Prove: if  $\pi$  in  $S_n$ , then  $\pi\alpha\pi^{-1}$  is the cycle

$$(\pi(a_1) \ \pi(a_2) \ \cdots \ \pi(a_j)).$$

4. Suppose that  $G, H$  are groups and  $\phi: G \rightarrow H$  is an isomorphism.

- (a) Prove: if every element of  $G$  is its own inverse, then every element of  $H$  is its own inverse.
- (b) Prove: if  $G$  is abelian, then  $H$  is abelian.

## References

[Cla] Our Group Theory Class, [Our Definition Sheet](#).