

# HW3

MTH 344

## Instructions

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

## 1 Computations

1. Write down every subgroup of  $\mathbb{Z}_5$ . (You can use “generator” notation. For example,  $\langle 1 \rangle = \{0, 1, 2, 3, 4\}$ .)
2. Write down every subgroup of  $\mathbb{Z}_{10}$ .
3. Write down every subgroup of  $\mathbb{Z}_{70}$ .
4. Do you have a conjecture about the number of subgroups of cyclic groups? (No need to turn in your answer to this question.)
5. How many surjective functions are there from  $\mathbb{Z}_2 \times \mathbb{Z}_2$  to  $\mathbb{Z}_2$ ? How many injective functions?
6. True/False:
  - (a)  $\mathbb{Q}$  is a subgroup of  $\mathbb{R}$ .
  - (b)  $\mathbb{Q}$  is a cyclic subgroup of  $\mathbb{R}$ .

## 2 Proofs

1. Let  $G$  be a group.

(a) Define

$$Z = \{g \in G \mid \text{for all } x \in G, xg = gx\}.$$

Prove that  $Z$  is a subgroup of  $G$ .

(b) Let  $S$  be any subset of  $G$  and define

$$C = \{g \in G \mid \text{for all } s \in S, sg = gs\}.$$

Prove that  $C$  is a subgroup of  $G$ .

2. Let  $G$  be a group, let  $H$  be a subgroup of  $G$ , and choose any  $g \in G$ . Let's use the notation

$$gHg^{-1} = \{ghg^{-1} \mid h \in H\}.$$

Prove  $gHg^{-1}$  is a subgroup of  $G$ .

3. Let  $A$  be a set and  $a \in A$ . Define

$$G = \{f \in S_A \mid f(a) = a\}.$$

Prove that  $G$  is a subgroup of  $S_A$ .