HW 4 Due: 4 November 2025, before I get to my office

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

1 Computations

1. Consider the function $g \in S_4$ given by

$$\begin{aligned} 1 &\mapsto 2 \\ 2 &\mapsto 1 \\ 3 &\mapsto 3 \\ 4 &\mapsto 4. \end{aligned}$$

   Write down all functions $f \in S_4$ with the property that

   (a) $f \circ f \circ f = \mathrm{id}_{\{1,2,3,4\}}$,

   (b) $f \circ g = g \circ f$,

   (c) $f \circ g = \mathrm{id}_{\{1,2,3,4\}}$.

   (d) $f(1) \in \{1,2\}$ and $f(2) \in \{1,2\}$.

2. How many elements (f,g) in $S_3 \times S_3$ are there that satisfy $(f,g)(f,g)(f,g) = \left(\mathrm{id}_{\{1,2,3\}}, \mathrm{id}_{\{1,2,3\}}\right)$?

2 Proofs

(I) Define the set

$$G = \{f \in S_5 \mid f(2) = 3\}.$$

Is G a subgroup of S_5? Prove your answer is correct.

(II) Suppose that n is a positive, and suppose that α is a nontrivial cycle in S_n. That is, there is a $j \in \{1,2,\ldots,n\}$ and distinct integers $a_1,\ldots,a_j$ such that

$$\alpha = (a_1 \ a_2 \ \cdots \ a_j).$$

Prove: if π in S_n, then $\pi\alpha\pi^{-1}$ is the cycle

$$(\pi(a_1) \ \ \pi(a_2) \ \ \cdots \ \pi(a_j)).$$