

Name: _____

- Put your name in the “ _____ ” above.
- Write your answers down neatly, use complete sentences, and *justify your work*.
- Good luck!

1. Let $b \in \mathbb{R}$ and

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ b+7 \\ b^2+b \end{bmatrix} \right\}.$$

For which value(s) of b is S a linearly dependent subset \mathbb{R}^3 ?

Solution. We must determine the real numbers b for which the following homogeneous system has a nontrivial solution:

$$\begin{bmatrix} 1 & 2 \\ 2 & b+7 \\ 3 & b^2+b \end{bmatrix} \mathbf{x} = \mathbf{0}.$$

To this end, we row reduce

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & b+7 & 0 \\ 3 & b^2+b & 0 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & b+3 & 0 \\ 0 & b^2+b-6 & 0 \end{array} \right].$$

This matrix has exactly one leading one when both

$$b+3=0 \quad \text{and} \quad b^2+b-6=0;$$

that is, when both

$$b+3=0 \quad \text{and} \quad (b+3)(b-2)=0.$$

Both equalities are satisfied exactly when $b = -3$, so

- S is linearly dependent when $b = -3$ and
- S is linearly independent when $b \in (-\infty, -3) \cup (-3, \infty)$.

□