Math 261 Spring 2025 Quiz 3

Name:_____

- Put your name in the "_____" above.
- Answer Question 1.
- Proofs are graded for correctness, clarity, rigor, neatness.
- Good luck!

1. Let
   - L be the line with direction vector $\begin{bmatrix}1\\2\\3\end{bmatrix}$ that contains the vector $\begin{bmatrix}1\\1\\1\end{bmatrix}$, and
   - M be the line with direction vector $\begin{bmatrix}1\\-1\\-1\end{bmatrix}$ that contains the vector $\begin{bmatrix}0\\8\\10\end{bmatrix}$.

   Do L and M intersect? If so, at which vector do they intersect?

   Solution. We must determine if there are $s, t \in \mathbb{R}$ such that

   $$s\begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix} = t\begin{bmatrix}1\\-1\\-1\end{bmatrix} + \begin{bmatrix}0\\8\\10\end{bmatrix}.$$

   Rearranging, we ask s, t to satisfy

   $$s\begin{bmatrix}1\\2\\3\end{bmatrix} + t\begin{bmatrix}-1\\1\\1\end{bmatrix} = \begin{bmatrix}-1\\7\\9\end{bmatrix}.$$

   Thus, we row reduce:

   $$\left[\begin{array}{cc|c}1&-1&-1\\2&1&7\\3&1&9\end{array}\right] \sim \left[\begin{array}{cc|c}1&-1&-1\\0&3&9\\0&4&12\end{array}\right] \sim \left[\begin{array}{cc|c}1&-1&-1\\0&1&3\\0&0&0\end{array}\right] \sim \left[\begin{array}{cc|c}1&0&2\\0&1&3\\0&0&0\end{array}\right]$$

   and we find the solution $(2, 3)$. Thus, we plug in 2 for s to see that the lines intersect at the vector

   $$2\begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}3\\5\\7\end{bmatrix}.$$

   To check our work, we could plug in 3 for t to find the same vector:

   $$3\begin{bmatrix}1\\-1\\-1\end{bmatrix} + \begin{bmatrix}0\\8\\10\end{bmatrix} = \begin{bmatrix}3\\5\\7\end{bmatrix}.$$

   □