Math 261 Spring 2025 Quiz 2

Name:_____

- Put your name in the "_____" above.
- Answer all questions.
- Proofs are graded for correctness, clarity, rigor, neatness.
- Good luck!

1. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 0 & 0 & 2 \\ -1 & 0 & 6 \\ -6 & -7 & -7 \end{bmatrix}.$$

Find all solutions to the matrix equation

$$(A + B)\,\mathbf{x} = \mathbf{0}.$$

Solution.

$$A + B = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 5 & 12 \\ 1 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & -3 \\ 0 & -1 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix},$$

so solutions to the equation are vectors of the form

$$t \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} \qquad \text{for any } t \in \mathbb{R}.$$

□

2. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 0 & 0 & 2 \\ -1 & 0 & 6 \\ -6 & -7 & -7 \end{bmatrix}.$$

Find all solutions to the matrix equation

$$A\mathbf{x} = -B\mathbf{x}.$$

Solution. Note that

$$A\mathbf{x} = -B\mathbf{x} \qquad \text{if and only if} \qquad (A + B)\mathbf{x} = \mathbf{0},$$

then proceed as above. □