

Name: _____

- Put your name in the “_____” above.
- Answer all questions.
- Proofs are graded for correctness, clarity, rigor, neatness.
- Good luck!

1. For which real numbers k does the following system have infinitely many solutions?

$$\begin{aligned}x + ky &= 0 \\(k - 1)x + 6y &= 0\end{aligned}$$

Solution. We begin to solve the augmented matrix:

$$\left[\begin{array}{cc|c} 1 & k & 0 \\ k-1 & 6 & 0 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & k & 0 \\ 0 & 6 - k(k-1) & 0 \end{array} \right],$$

so we see this system has infinitely many solutions when

$$0 = 6 - k(k-1) = 6 + k - k^2 = -(k-3)(k+2).$$

That is, the system has infinitely many solutions when $k \in \{3, -2\}$.

□