## Math 261

Name:\_\_\_\_\_

- Put your name in the "\_\_\_\_\_" above.
- Answer all questions.
- Solutions are graded for correctness, clarity, rigor, neatness.
- Good luck!
- 1. Let

	[1	2	3]			[2]	
<i>A</i> =	4	5	6	and	b =	5	
	7	8	9		b =	8	

Writing  $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ , find all solutions to the matrix equation

 $A\mathbf{x} = \mathbf{b}.$ 

2. Suppose that

- B is a  $2 \times 3$  matrix,
- C is a  $3 \times 2$  matrix,
- D is a  $3 \times 3$  matrix, and
- E is a  $2 \times 1$  matrix.

For each of the following matrix expressions, either tell me the size of the matrix or write "Undefined."

(a) BC

(b) CB

(c) B + C

(d) C + B

(e) *BD* 

(f) BE

(g) BDC

- 3. Define a function  $T: \mathbb{R}^2 \to \mathbb{R}^2$  by letting  $T(\mathbf{x})$  be the vector obtained by rotating  $\mathbf{x}$  counterclockwise by 270° (which is the same as  $\frac{3\pi}{2}$  radians).
  - (a) Compute

$$T\left(\begin{bmatrix}1\\2\end{bmatrix}\right)$$

(b) You may assume that T is a linear transformation. Find a matrix F such that for all  $\mathbf{x} \in \mathbb{R}^2$ ,

 $T(\mathbf{x}) = F\mathbf{x}.$ 

4. Let

$$G = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 6 \\ 1 & 1 & 2 \end{bmatrix}.$$

Find  $G^{-1}$ , if it exists.

5. (a) Write a system of two linear equations in two variables that has infinitely many solutions.

(b) Solve your system from part (a).

6. Let

$$H = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(a) Let  $E_1$  be the elementary matrix associated to scaling row two of H by 2. What is  $E_1$ ?

- (b) Let  $E_2$  be the elementary matrix associated to adding  $(-1) \cdot (row \text{ one})$  to row two. What is  $E_2$ ?
- (c) Let  $E_3$  be the elementary matrix associated to adding  $(-1) \cdot (row three)$  to row two. What is  $E_3$ ?

(d) What is  $E_3E_2E_1H$ ?

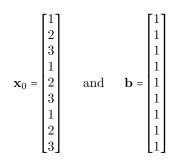
7. Suppose that a is a real number, and let

$$J = \begin{bmatrix} a & 2a \\ 3a & 4a+2 \end{bmatrix}.$$

For which values of a is J invertible? (Hint: first consider the case where a = 0, then consider all other cases.)

## Extra credit

Let



Write down a  $9\times9$  matrix J such that

 $J\mathbf{x} = \mathbf{b}$  has infinitely many solutions and  $J\mathbf{x} = \mathbf{b}$  has  $\mathbf{x}_0$  as a solution.