Math 261 Practice Quiz 4

Name:____

- Put your name in the "____" above.
- Write your answers down neatly, use complete sentences, and *justify your work.*
- Good luck!

1. Let

$$V = \operatorname{Span}\left\{\begin{bmatrix}6\\1\\1\\5\end{bmatrix}, \begin{bmatrix}17\\3\\2\\10\end{bmatrix}, \begin{bmatrix}52\\9\\7\\35\end{bmatrix}, \begin{bmatrix}18\\3\\4\\20\end{bmatrix}\right\}.$$

(a) What is a basis for V?

(b) What is the dimension of V?

Solution. We row reduce the appropriate matrix:

$$\begin{bmatrix}6 & 17 & 52 & 18\\1 & 3 & 9 & 3\\1 & 2 & 7 & 4\\5 & 10 & 35 & 20\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 7 & 4\\1 & 3 & 9 & 3\\6 & 17 & 52 & 18\\5 & 10 & 35 & 20\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 7 & 4\\0 & 1 & 2 & -1\\0 & 5 & 10 & -6\\0 & 0 & 0 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 7 & 4\\0 & 1 & 2 & -1\\0 & 0 & 0 & -1\\0 & 0 & 0 & 0\end{bmatrix}.$$

Since columns 1, 2, and 4 contain leading ones, we see that

(a) a basis for V is $\left\{\begin{bmatrix}6\\1\\1\\5\end{bmatrix}, \begin{bmatrix}17\\3\\2\\10\end{bmatrix}, \begin{bmatrix}18\\3\\4\\20\end{bmatrix}\right\}$ and

(b) the dimension of V is 3.

□

2. Let $b \in \mathbb{R}$ and

$$W = \operatorname{Span}\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\b+7\\b^2+b\end{bmatrix}\right\}.$$

For which value(s) of b is W a 1-dimensional subspace of $\mathbb{R}^3$?

Solution. To find a basis for W, we row reduce the appropriate matrix:

$$\begin{bmatrix}1 & 2\\2 & b+7\\3 & b^2+b\end{bmatrix} \sim \begin{bmatrix}1 & 2\\0 & b+3\\0 & b^2+b-6\end{bmatrix}.$$

This matrix has one leading one pivot precisely when both

$$b+3=0 \qquad \text{and} \qquad b^2+b-6=0;$$

that is, when both

$$b+3=0 \qquad \text{and} \qquad (b+3)(b-2)=0.$$

Both equalities are satisfied only when $b=-3$, so

- W has dimension 1 when $b=-3$ and
- W has dimension 2 when $b \in (-\infty,-3) \cup (-3,\infty)$.

□