

Math 261 Section 6 Fall 2019 Exam 2 November 20, 2019

Name: _____

- Put your name in the “ _____ ” above.
- Write your answers down neatly, use complete sentences, and *justify your work*.
- The extra credit question is for your amusement, if you have extra time at the end of the test.
- Good luck!

1. Let

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \text{ and } \mathbf{w} = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}.$$

If possible, write \mathbf{w} as a linear combination of \mathbf{u} and \mathbf{v} .

2. Suppose that a is a real number, and let

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & -1 & -3 & -4 \\ 1 & 2 & a+5 & 4 \\ 2 & 4 & 6 & a+6 \end{pmatrix}.$$

For which value(s) of a is A invertible?

3. Suppose that

- L_1 be the line through the points $(1, 2, 3)$ and $(5, 6, 7)$ and
- L_2 be the line between $(1, 6, 2)$ and $(-7, 2, -7)$.

(a) What is the vector equation of L_1 ?

(b) What is the vector equation of L_2 ?

(c) Do these lines intersect? If so, where do they intersect?

4. Let

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}.$$

Find two vectors (let's call them \mathbf{u}_{\parallel} and \mathbf{u}_{\perp}) such that

- \mathbf{u}_{\parallel} is a scalar multiple of \mathbf{v} ,
- \mathbf{u}_{\perp} is perpendicular to \mathbf{v} , and
- $\mathbf{u} = \mathbf{u}_{\parallel} + \mathbf{u}_{\perp}$.

HINT: this is a projection question—recall that $\text{proj}_{\mathbf{v}}(\mathbf{u})$ is a scalar multiple of \mathbf{v} .

5. Let

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

Find a vector \mathbf{v} such that

- The first coordinate of \mathbf{v} is 7 and
- \mathbf{v} is perpendicular to \mathbf{u} .

6. Suppose that b be a real number and let

$$\mathbf{u} = \begin{bmatrix} b \\ -2b+4 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \text{ and } \mathbf{w} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}.$$

For which value(s) of b is \mathbf{u} in $\text{Span}\{\mathbf{v}, \mathbf{w}\}$?

Extra Credit

Let

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

Find a vector \mathbf{v} such that

- The first coordinate of \mathbf{v} is 7,
- \mathbf{v} is perpendicular to \mathbf{u} , and
- $\|\mathbf{v}\| = \sqrt{83}$.