Math 261 Section 6 Fall 2019 Exam 1 October 23, 2019

Name:____

- Put your name in the "____" above.
- Write your answers down neatly, use complete sentences, and *justify your work.*
- Good luck!

1. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}.$$

Writing $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, find all solutions to the matrix equation

$$A\mathbf{x} = \mathbf{b}.$$

2. Suppose that

- B is a 2×3 matrix,
- C is a 3×2 matrix,
- D is a 3×3 matrix, and
- E is a 2×1 matrix.

For each of the following matrix expressions, either tell me the size of the matrix or write "Undefined."

(a) BC

(b) CB

(c) $B + C$

(d) $C + B$

(e) BD

(f) BE

(g) BDC

3. Let

$$F = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 6 \\ 1 & 1 & 2 \end{bmatrix}.$$

Find F^{-1}, if it exists.

4. (a) Write a system of two linear equations in two variables that has infinitely many solutions.

(b) Solve your system from part (a).

5. Suppose that a is a real number, and let

$$G = \begin{bmatrix} a & 2a \\ 3a & 4a + 2 \end{bmatrix}.$$

For which values of a is G invertible?

6. Let

$$H = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(a) Let E_1 be the elementary matrix associated to scaling row two of H by 2. What is E_1?

(b) Let E_2 be the elementary matrix associated to adding $(-1) \cdot$ (row one) to row two. What is E_2?

(c) Let E_3 be the elementary matrix associated to adding $(-1) \cdot$ (row three) to row two. What is E_3?

(d) What is $E_3 E_2 E_1 H$?

Extra Credit

Let

$$\mathbf{x}_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \\ 2 \\ 3 \\ 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Write down a 9×9 matrix J such that

$$J\mathbf{x} = \mathbf{b} \text{ has infinitely many solutions} \quad \text{and} \quad J\mathbf{x} = \mathbf{b} \text{ has } \mathbf{x}_0 \text{ as a solution.}$$