Math 261 Fall 2024 Practice Quiz 2

Name:_____

- Put your name in the "_____" above.
- Answer all questions.
- Proofs are graded for correctness, clarity, rigor, neatness.
- Good luck!

1. For which real numbers k is the following matrix invertible?

$$A = \begin{bmatrix} 1 & k \\ k-1 & 6 \end{bmatrix}$$

Solution. A matrix is invertible if and only if it is row-reducible to the identity matrix. Compute

$$\begin{bmatrix} 1 & k \\ k-1 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & k \\ 0 & 6-k(k-1) \end{bmatrix}$$

to see that A is *not* row reducible to the identity if and only if

$$0 = 6 - k(k-1) = 6 + k - k^2 = -(k-3)(k+2).$$

Thus, we see that A is not invertible if k is either 3 or -2, and it is invertible for all other real numbers. □