Math 261 Fall 2024 Exam One

Name:_____

- Put your name in the "_____" above.

- Answer all questions.

- Solutions are graded for correctness, clarity, rigor, neatness.

- Good luck!

1. Solve the following system of linear equations.

$$2x + y + z = 1$$
$$3x + 2y + 3z = -2$$
$$4x + 3y + 5z = -5.$$

2. Define a function $T: \mathbb{R}^3 \to \mathbb{R}^3$ by the rule

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y + z \\ z \\ y \end{bmatrix}.$$

(a) Compute

$$T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right).$$

(b) You may assume that T is a linear transformation. Find a matrix A such that for all $\mathbf{x} \in \mathbb{R}^3$,

$$T(\mathbf{x}) = A\mathbf{x}.$$

(c) Find all vectors $\mathbf{u}$ with the property that

$$T(\mathbf{u}) = \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix}.$$

3. (a) Write a system of three linear equations in three variables that has infinitely many solutions.

   (b) Solve your system from part (a).

4. Suppose that A, B, C are matrices.

(a) Suppose that A is 3×8, that C is 3×15, and that $AB = C$. What is the size of B?

(b) Suppose that A is 10×10. What is the size of A^{99}?

(c) Suppose that A is 1×2, that B is 2×3 and C is 3×4. What is the size of ABC?

5. For which real numbers a is the following matrix invertible?

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & a & 2 \\ 0 & 3 & a \end{bmatrix}$$

6. Write the following matrix as a product of elementary matrices. (You don't need to write all the 0s in your matrices.)

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

7. Let a be a real number,

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix}.$$
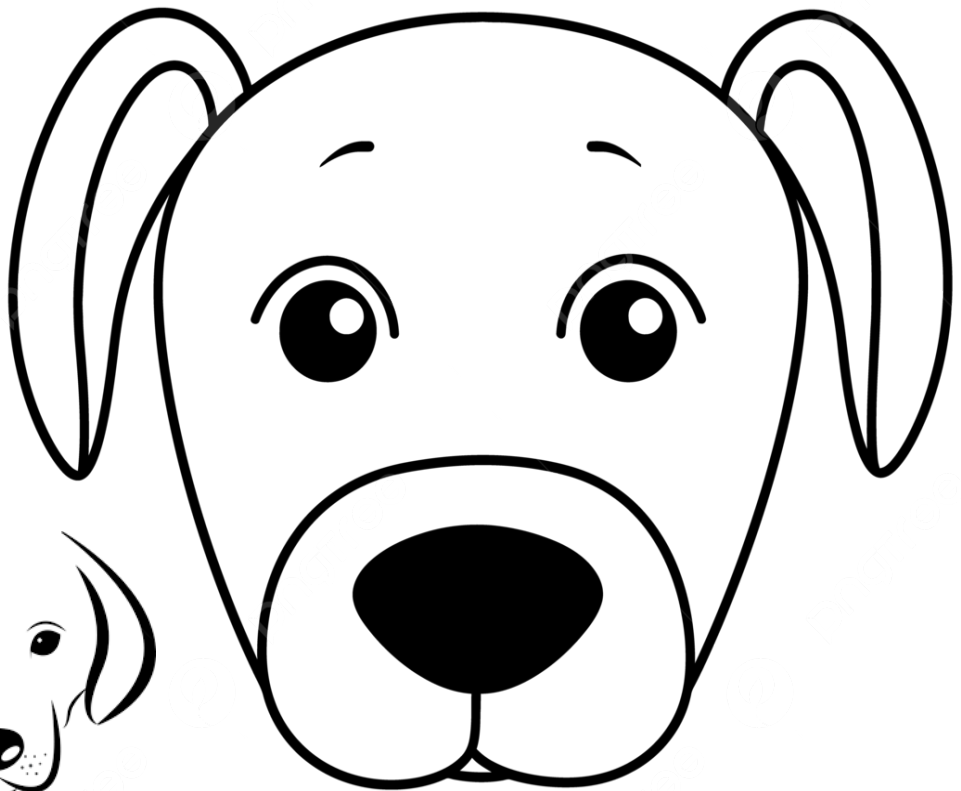
For which values of a is $\mathbf{w}$ a linear combination of $\mathbf{u}$ and $\mathbf{v}$?

Extra credit

Suppose that Adira, Badele, and Connor are three cute dogs. We know that

- Badele weighs twice as much as Adira,

- Connor's weight is the sum of Adira's weight and Badele's weight, and

- Connor weighs 80 pounds more than Adira.

Name the dogs based on their size.

name:_____   name:_____   name:_____