

Mixed $2^k \times 3^k$ designs

Stat 526
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Example: $2^1 \times 3^1$ design

①

1 factor with 2 levels: A $\rightarrow +$

1 factor with 3 levels: X x_1, x_2, x_3

Scheme: replace the "3" with " 2^2 "

This becomes a $2^1 \times 2^2 = 2^3$ design

A	B	C	X
-	-	-	x_1
-	-	+	x_2
-	+	-	x_2
-	+	+	x_3
+	-	-	x_1
+	-	+	x_2
+	+	-	x_2
+	+	+	x_3

A	X
Low	Low
Low	Mid
Low	Mid
Low	High
High	Low
High	Mid
High	Mid
High	High

②

mapping of (B, C) $\rightarrow$ X :

$(-, -) \rightarrow x_1$
 $(+, +) \rightarrow x_3$
 $(+, -) \rightarrow x_2$
 $(-, +) \rightarrow x_2$

Source	df
A	1
X	2
A X	2
Error	2
Total	7

③
Recall our section on orthogonal polynomials. Since X has 2 df, its sum of squares can be decomposed into a linear portion, a quadratic portion, with 1 df each.

Source	df
A	1
X_L	1
X_Q	1
$A \times X_L$	1
$A \times X_Q$	1
Err	2
Total	7

④
Class project:

$2^1 \times 3^2$ design

First: create a

$2^1 \times (2^2)^2 = 2^5$ design
(32 runs)

A
 B
 C
 D
 E
 X
 W

A
 Low
 X
 W
 7
 duplicate
 Then repeat with
 A high

Source	df	
A	1	
X	2	X_L, X_Q
W	2	W_L, W_Q
AX	2	$A \times X_L, A \times X_Q$
AW	2	$A \times W_L, A \times W_Q$
XW	4	$X_L \times W_L, X_L \times W_Q, X_Q \times W_L, X_Q \times W_Q$
AXW	4	$A \times \text{each of these } \uparrow$
Err	14	
Tot	31	

Example: Mixed 2^k & 4^k

(7)

Scheme: replace the "4" with " 2^2 "

Consider 2 2-level factors and
1 4-level factor.

$$2^2 \times 4^1 \Rightarrow 2^2 \times 2^2 = 2^4 \text{ design}$$

(16 runs)

A	B	C	D	X
-	-	-	-	x_1
-	-	-	+	x_2
-	-	+	-	x_3
-	-	+	+	x_4
-	+	-	-	x_1
-	+	-	+	x_2
-	+	+	-	x_3
-	+	+	+	x_4
+	-	-	-	x_1
+	-	-	+	x_2
+	-	+	-	x_3
+	-	+	+	x_4
+	+	-	-	x_1
+	+	-	+	x_2
+	+	+	-	x_3
+	+	+	+	x_4

(8)

$$(C, D) \rightarrow X$$

$$(-, -) \rightarrow x_1$$

$$(-, +) \rightarrow x_2$$

$$(+, -) \rightarrow x_3$$

$$(+, +) \rightarrow x_4$$

⑨

Source	df	
A	1	SS_A
B	1	SS_B
X	3	$SS_C + SS_D + SS_{CD}$
AB	1	SS_{AB}
AX	3	$SS_{AC} + SS_{AD} + SS_{ACD}$
BX	3	$SS_{BC} + SS_{BD} + SS_{BCD}$
ABX	3	$SS_{ABC} + SS_{ABD} + SS_{ABCD}$
Error	0	
Total	15	