Randomization restriction example

Stat 5606
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(1)

Oxide growth on silicon wafer

A: temperature 2 levels

B: gas 2 levels

C: time 2 levels

D: wafer position in furnace 2 levels

All fixed, all crossed

2 replications (32 total runs)

But...

(2)

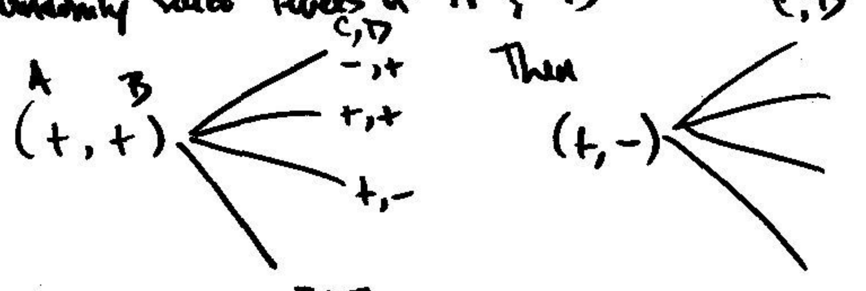
Restrictions:

They can only do 16 runs per day,

So they will run 1 entire replication of the experiment on each day

Day 1

Randomly select levels of A & B



repeat for (−,−), (−,+)

Repeat process on day 2

(3)

Model:

$$
\begin{array}{ccccccc}
 & & T & A & B & C & D \\
 & & \text{day} & \text{temp} & \text{gas} & \text{time} & \text{position} \\
 & & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
y_{ijklmn} = & \mu + & \tau_i + & \beta_j + & \gamma_k + & \delta_l + & \lambda_m +
\end{array}
$$

10 2-way interactions +

10 3-way interactions +

5 4-way interactions +

1 5-way interaction + $\varepsilon_{(ijklm)n}$

When working out the EMS terms, we can omit T and any interactions involving T, because their F-test denominators will be MSE and $df_E = 0$

(4)

	R i	F j	F k	F l	F m	R n	EMS
τ_i	1	2	2	2	2	1	$16\sigma_\tau^2 + \sigma^2$
β_j	2	0	2	2	2	1	.
γ_k	2	2	0	2	2	1	.
δ_l	2	2	2	0	2	1	.
λ_m	2	2	2	2	0	1	.
$\beta\gamma_{jk}$	2	0	0	2	2	1	as usual
$\beta\delta_{jl}$	2	0	2	0	2	1	
$\beta\lambda_{jm}$	2	0	2	2	0	1	
$\gamma\delta_{kl}$	2	2	0	0	2	1	
$\gamma\lambda_{km}$	2	2	0	2	0	1	
$\delta\lambda_{lm}$	2	2	2	0	0	1	
$\varepsilon_{(ijklm)n}$	1	1	1	1	1	1	

5

Source	df
Blocks	1
A	1
B	1
TA	1
TB	1
TAB	1
C	
D	
CD	
+ all others	
error	0
	/31

$\left.\begin{matrix}A\\B\\TA\\TB\\TAB\end{matrix}\right\}$ whole plot 6 df

$\left.\begin{matrix}C\\D\\CD\\+\text{ all others}\end{matrix}\right\}$ subplot 24 df

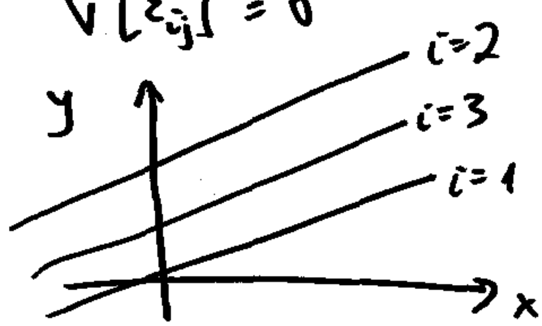
6

Analysis of Covariance (ANOCOVA)

$$y_{ij} = \mu + \tau_i + \beta(x_{ij} - \bar{x}_{..}) + \varepsilon_{ij} \qquad \begin{matrix} i = 1, \dots, a \\ j = 1, \dots, n \end{matrix}$$

$$\sum_{i=1}^{a} \tau_i = 0 \qquad E[\varepsilon_{ij}] \equiv 0$$

$$V[\varepsilon_{ij}] \equiv \sigma^2, \quad \varepsilon_{ij} \text{ indep } \forall i,j$$



(7)

Notation:

$$S_{yy} = \sum_i \sum_j (y_{ij} - \bar{y}_{..})^2 = \sum_i \sum_j y_{ij}^2 - an\bar{y}_{..}^2$$

$$S_{xx} = \sum_i \sum_j (x_{ij} - \bar{x}_{..})^2 = \quad \ldots$$

$$S_{xy} = \sum_i \sum_j (x_{ij} - \bar{x}_{..})(y_{ij} - \bar{y}_{..}) = \quad \ldots$$

$$T_{yy} = n \sum_i (\bar{y}_{i.} - \bar{y}_{..})^2$$

$$T_{xx} = n \sum_i (\bar{x}_{i.} - \bar{x}_{..})^2$$

(8)

$$T_{xy} = n \sum_i (\bar{x}_{i.} - \bar{x}_{..})(\bar{y}_{i.} - \bar{y}_{..})$$

$$E_{yy} = \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2$$

$$E_{xx} = \sum_i \sum_j (x_{ij} - \bar{x}_{i.})^2$$

$$E_{xy} = \sum_i \sum_j (x_{ij} - \bar{x}_{i.})(y_{ij} - \bar{y}_{i.})$$

Goal: Estimate all of the parameters and test $H_0: \tau_i \equiv 0$

$$SSE = \sum_i \sum_j \left[y_{ij} - \mu - \tau_i - \beta(x_{ij} - \bar{x}_{..}) \right]^2$$

$$\frac{\partial SSE}{\partial \mu} = \sum_i \sum_j \cancel{2}\left[y_{ij} - \mu - \tau_i - \beta(x_{ij} - \bar{x}_{..}) \right](-\cancel{1}) \stackrel{set}{=} 0$$

$$y_{..} - an\mu - 0 - \beta(\cancel{x_{..}} - \cancel{an\bar{x}_{..}}) = 0$$

$$\hat{\mu} = \frac{y_{..}}{an} = \bar{y}_{..}$$

$$\frac{\partial SSE}{\partial \tau_i} = \sum_j \cancel{2}\left[y_{ij} - \mu - \tau_i - \beta(x_{ij} - \bar{x}_{..}) \right](-\cancel{1}) \stackrel{set}{=} 0$$

$$y_{i.} - n\mu - n\tau_i - \beta(x_{i.} - n\bar{x}_{..}) = 0$$

10

$$\hat{\tau}_i = \bar{y}_{i.} - \bar{y}_{..} - \hat{\beta}(\bar{x}_{i.} - \bar{x}_{..})$$

$$\frac{\partial SSE}{\partial \beta} = \sum_i \sum_j \cancel{2}\left[y_{ij} - \mu - \tau_i - \beta(x_{ij} - \bar{x}_{..}) \right](-\cancel{1})(x_{ij} - \bar{x}_{..}) \stackrel{set}{=} 0$$

$$\sum_i \sum_j (y_{ij} - \bar{y}_{..})(x_{ij} - \bar{x}_{..}) - \sum_i \sum_j \hat{\tau}_i (x_{ij} - \bar{x}_{..}) - \hat{\beta} \sum_i \sum_j (x_{ij} - \bar{x}_{..})^2 = 0$$

$$S_{xy} - \sum_i \sum_j \hat{\tau}_i (x_{ij} - \bar{x}_{..}) - \hat{\beta} S_{xx} = 0 \qquad (11)$$

to be continued