Randomization restrictions

Stat 526
4-17-25

Example: Paper manufacturing

Factor A : pulp preparation method
3 levels, fixed

Factor B: cooking temperature
4 levels, fixed

Run 3 replications,

so there are $3 \times 4 \times 3 = 36$ runs,

but...

To run it as a complete randomized design, the 36 runs must be done in random order.
They can only do 12 runs per day.

To run it as a randomized complete block design,
the 12 runs on each day must be done in a
random order.
but...

Each day, they randomly select a pulp preparation
method, + prepare a batch of pulp
They divide it into 4 equal parts + cook
each part at a different temperature.
Repeat for the other 2 preparation methods.

(3)

Model $y_{ijkl} = \tau_i + \beta_j + \gamma_k + \tau\beta_{ij} + \tau\gamma_{ik} + \beta\gamma_{jk} + \tau\beta\gamma_{ijk} + \varepsilon_{(ijk)l}$ (day, prep, temp)

	R i	F j	F k	t_R l	EMS
τ_i	1	3	4	1	$12\sigma^2_\tau + \sigma^2$
β_j	3	0	4	1	$12\sum\beta_j^2/2 + 4\sigma^2_{\tau\beta} + \sigma^2$
γ_k	3	3	0	1	$9\sum\gamma_k^2/3 + 3\sigma^2_{\tau\gamma} + \sigma^2$
$\tau\beta_{ij}$	1	0	4	1	$4\sigma^2_{\tau\beta} + \sigma^2$
$\tau\gamma_{ik}$	1	3	0	1	$3\sigma^2_{\tau\gamma} + \sigma^2$
$\beta\gamma_{jk}$	3	0	0	1	$3\sum\sum(\beta\gamma)^2_{jk}/6 + \sigma^2_{\tau\beta\gamma} + \sigma^2$
$\tau\beta\gamma_{ijk}$	1	0	0	1	$\sigma^2_{\tau\beta\gamma} + \sigma^2$
$\varepsilon_{(ijk)l}$	1	1	1	1	σ^2

(4)

Source	df	denom of F test
~~Day~~	~~2~~	~~MS_E~~
Prep	2	$MS_{Day \times Prep}$
Temp	3	$MS_{Day \times temp}$
~~Day x Prep~~	~~4~~	~~MS_E~~
~~Day x Temp~~	6	~~MS_E~~
Prep x Temp	6	$MS_{Day \times Prep \times temp}$
~~Day x Prep x Temp~~	12	~~MS_E~~
Error	0	

(5)

Different layout of the ANOVA table (Split plot)

Source	df	
Days (blocks)	2	
Prep (A)	2	whole plot
Days × Prep	4	
Temp (B)	3	whole plot
Days × Temp	6	
A B	6	sub plot
Days × Temp × Prep	12	
	35	

(6)

Revisit the RCBD

$i = 1, \ldots, a$

$j = 1, \ldots, b$

$k = 1$

$$y = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \varepsilon_{(ij)k}$$

	F i	R j	R k	EMS
τ_i	0	b	1	$b\frac{\sum \tau_i^2}{a-1} + \sigma^2_{\tau\beta} + \sigma^2$
β_j	a	1	1	$a\sigma^2_\beta + \sigma^2$
$\tau\beta_{ij}$	0	1	1	$\sigma^2_{\tau\beta} + \sigma^2$
$\varepsilon_{(ij)k}$	1	1	1	σ^2

Source	df	denom for F
A	$a-1$	MS_{AB}
~~B~~	~~$b-1$~~	~~MS_E~~
~~AB~~	~~$(a-1)(b-1)$~~	~~MS_E~~
Error	0	

Split plot version

Source	df
B	$b-1$
A (whole plot)	$a-1$
AB (whole plot)	$(a-1)(b-1)$
	$ab-1$

$b-1+a-1+ab-a-b+1$

HW #3 14.2, 14.7

14.2. The surface finish of metal parts made on four machines is being studied. An experiment is conducted in which each machine is run by three different operators and two specimens from each operator are collected and tested. Because of the location of the machines, different operators are used on each machine, and the operators are chosen at random. The data are shown in the following table. Analyze the data and draw conclusions.

	Machine 1			**Machine 2**			**Machine 3**			**Machine 4**		
Operator	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
	79	94	46	92	85	76	88	53	46	36	40	62
	62	74	57	99	79	68	75	56	57	53	56	47

14.7. Derive the expected mean squares for a balanced three-stage nested design, assuming that A is fixed and that B and C are random. Obtain formulas for estimating the variance components. Assume the restricted form of the mixed model.