Another nested/crossed example

Stat 506
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(1)

We wish to see if output is affected by workplace layout, assembly fixture, and operator.

We will study 2 particular layouts.

For each layout, 3 different assembly fixtures are used.

For each layout, 4 operators are selected from a larger group.

2 replicates are run.

(2)

A: layout fixed $a=2$

B(A): fixture fixed $b=3$

C(A): operator random $c=4$

$n=2$

$$y_{ijkl} = \mu + \tau_i + \beta_{j(i)} + \gamma_{k(i)} + \beta\gamma_{jk(i)} + \varepsilon_{(ijk)l}$$

(3)

	$\overset{2}{F}$ i	$\overset{3}{F}$ j	$\overset{4}{R}$ k	$\overset{2}{R}$ l	EMS
τ_i	0	3	4	2	$24\frac{\sum\tau_i^2}{1} + 6\sigma_\gamma^2 + \sigma^2$
$\beta_{j(i)}$	1	0	4	2	$8\frac{\sum\sum\beta_{j(i)}^2}{4} + 2\sigma_{\beta\gamma}^2 + \sigma^2$
$\gamma_{k(i)}$	1	3	1	2	$6\sigma_\gamma^2 + \sigma^2$
$\beta\gamma_{jk(i)}$	1	0	1	2	$2\sigma_{\beta\gamma}^2 + \sigma^2$
$\varepsilon_{(ijk)l}$	1	1	1	1	σ^2

(4)

Source	df	F
A	1	MSA/MSC(A)
B(A)	4	MSB(A)/MSBC(A)
C(A)	6	MSC(A)/MSE
BC(A)	12	MSBC(A)/MSE
Error	24	
Total	47	

A manufacturer wishes to know if operator (A), material (B), and heat (C) affect the outcome of the product being produced. All of the machines being used operate at the same four heat settings. Five operators are randomly chosen. There are fifteen varieties of material that can be used, and three of these are assigned to each of the five operators. For each operator, that means that there are 12 combinations of heat and material that can be used. The operator produces two replications of each of these, for a total of $5 \times 3 \times 4 \times 2 = 120$ observations.

Incorrectly treating all main effects as fixed and all combinations of factors as crossed, the statistician produces the ANOVA table shown below.

Source	SS	df	MS	F	pval < 0.05
A	34	4	8.5	3.70	*
B	12	2	6.0	2.61	
C	24	3	8.0	3.48	*
AB	32	8	4.0	1.74	
AC	30	12	2.5	1.09	
BC	18	6	3.0	1.30	
ABC	36	24	1.5	0.65	
Error	138	60	2.3		
Total	324	119			

Produce the corrected ANOVA table, including the expected mean squares and the correct F statistics. If an exact F test is not available, construct an approximate F statistic (for which you need not compute the degrees of freedom).

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A: operator random $a=5$

B: material fixed $b=3$ nested within A

C: heat fixed $c=4$

$n=2$

$$y_{ijkl} = \mu + \tau_i + \beta_{j(i)} + \gamma_k + \tau\gamma_{ik} + \beta\gamma_{jk(i)} + \varepsilon_{(ijk)l}$$

(7)

	5 R i	3 F j	4 F k	2 R l	EMS	den
τ_i	1	3	4	2	$24\sigma_\tau^2 + \sigma^2$	MSE
$\beta_{j(i)}$	1	0	4	2	$8\sigma_\beta^2 + \sigma^2$	MSE
γ_k	5	3	0	2	$30\,\frac{\sum \gamma_k^2}{3} + 6\sigma_{\tau\gamma}^2 + \sigma^2$	MSAC
$\tau\gamma_{ik}$	1	3	0	2	$6\sigma_{\tau\gamma}^2 + \sigma^2$	MS_E
$\beta\gamma_{jk(i)}$	1	0	0	2	$2\sigma_{\beta\gamma}^2 + \sigma^2$	MS_E
$\varepsilon_{(ijk)l}$	1	1	1	1	σ^2	

(8)

Source	SS	df	MS	F	p-val	
A	34	4	8.5	3.70	.009	*
B(A)	44	10	4.4	1.91	.06	
C	24	3	8	3.2	.06	
AC	30	12	2.5	1.09	.385	
BC(A)	54	30	1.8	0.78	.768	
Err	138	60	2.3			
TOT	324	119				