Rules for finding
expected mean squares (restricted mixed version) ①

① Write $\varepsilon_{ijkl} = \varepsilon_{(ijk)l}$

② If a term has a subscript in parentheses, then there will be no interactions between the factors represented by indices in ( ) and those outside.

Example: $\beta_{j(i)} \Rightarrow$ no AB interaction

③ Subscripts outside the ( ) are live
" inside the ( ) are dead
" not present in the term are absent

④ For a random effect, the variance component is of the form σ^2_z

For a fixed effect, the variance component is of the form $\frac{\sum_{i=1}^{a} \tau_i^2}{a-1}$

EMS table (Example)

Factor	F i	R j	R k	EMS
τ_i				
β_j				
$\tau\beta_{ij}$				
$\varepsilon_{(ij)k}$				

(5) (a) Enter a "1" in the table for each dead subscript in that term

(b) Enter a "0" for a live subscript in an F column & a "1" " " " " " "R"

(c) For absent subscripts, enter the number of levels of that factor

Complete parts (a), (b), (c) for last Thursday's example (A fixed, B random)

Factor	F i	R j	R k	EMS	F
τ_i	0	b	n	$bn\frac{\sum\tau_i^2}{a-1} + n\sigma^2_{\tau\beta} + \sigma^2$	MS_A/MS_{AB}
β_j	a	1	n	$an\sigma^2_\beta + \sigma^2$	MS_B/MS_E
$\tau\beta_{ij}$	0	1	n	$n\sigma^2_{\tau\beta} + \sigma^2$	MS_{AB}/MS_E
$\varepsilon_{(ij)k}$	1	1	1	σ^2	

(d) In each row, cover all of the columns corresponding to the Roman subscripts for that term. Then the EMS will contain terms corresponding to each row with at least the same subscripts, and the coefficient of the variance is the product of the visible numbers.

(6) For each H_0, find a term whose EMS matches the EMS of the numerator, under H_0. Use that term as the denominator

Example: 3 factors: A fixed, B & C random

	F	R	R	R	
Factor	i	j	k	l	EMS
τ_i	0	b	c	n	$bcn\frac{\sum \tau_i^2}{a-1} + cn\sigma^2_{\tau\beta} + bn\sigma^2_{\tau\gamma} + n\sigma^2_{\tau\beta\gamma} + \sigma^2$
β_j	a	1	c	n	$acn\sigma^2_\beta + an\sigma^2_{\beta\gamma} + \sigma^2$
γ_k	a	b	1	n	$abn\sigma^2_\gamma + an\sigma^2_{\beta\gamma} + \sigma^2$
$\tau\beta_{ij}$	0	1	c	n	$cn\sigma^2_{\tau\beta} + n\sigma^2_{\tau\beta\gamma} + \sigma^2$
$\tau\gamma_{ik}$	0	b	1	n	$bn\sigma^2_{\tau\gamma} + n\sigma^2_{\tau\beta\gamma} + \sigma^2$
$\beta\gamma_{jk}$	a	1	1	n	$an\sigma^2_{\beta\gamma} + \sigma^2$
$\tau\beta\gamma_{ijk}$	0	1	1	n	$n\sigma^2_{\tau\beta\gamma} + \sigma^2$
$\varepsilon_{(ijk)l}$	1	1	1	1	σ^2

F test

A: no exact F test ← next time, we'll see an approximate F test

B: MS_B / MS_{BC}

C: MS_C / MS_{BC}

AB: MS_{AB} / MS_{ABC}

AC: MS_{AC} / MS_{ABC}

BC: MS_{BC} / MS_E

ABC: MS_{ABC} / MS_E

Rule for df:

df = product of (#levels - 1) for each love subscript times (#levels) for each dead subscript

In our last example,

$df_{AB} = (a-1)(b-1)$

$df_E = abc(n-1)$

Rule for find SS terms:

Start by symbolically multiplying the df

AB: $df = (a-1)(b-1) = ab - a - b + 1$

For each term in the expansion, put a dot for each missing index, square the term, divide by the #levels of each missing index, & sum over all present indices.

$$SS_{AB} = \sum_i \sum_j \frac{y_{ij\cdot\cdot}^2}{cn} - \sum_i \frac{y_{i\cdot\cdot\cdot}^2}{bcn} - \sum_j \frac{y_{\cdot j\cdot\cdot}^2}{acn} + \frac{y_{\cdot\cdot\cdot\cdot}^2}{abcn}$$

A, B Axed C random

Factor	F i	F j	R k	R l	EMS	denom
τ_i	0	b	c	n	$bcn\,\Sigma\tau_i^2/(a-1) + bn\sigma^2_{\tau\beta} + \sigma^2$	AC
β_j	a	0	c	n	$acn\,\Sigma\beta_j^2/(b-1) + an\sigma^2_{\beta\gamma} + \sigma^2$	BC
γ_k	a	b	1	n	$ab n\sigma^2_\gamma + \sigma^2$	ERR
$\tau\beta_{ij}$	0	0	c	n	$cn\,\frac{\Sigma\Sigma(\tau\beta)_{ij}^2}{(a-1)(b-1)} + n\sigma^2_{\tau\beta\gamma} + \sigma^2$	ABC
$\tau\gamma_{ik}$	0	b	1	n	$bn\sigma^2_{\tau\beta\gamma} + \sigma^2$	ERR
$\beta\gamma_{jk}$	a	0	1	n	$an\sigma^2_{\beta\gamma} + \sigma^2$	ERR
$\tau\beta\gamma_{ijk}$	0	0	1	n	$n\sigma^2_{\tau\beta\gamma} + \sigma^2$	ERR
$\varepsilon_{(ijk)l}$	1	1	1	1	σ^2	