Select a particular index k where $P(C_k) \neq 0$ ①

$$P(C_k | A) = \frac{P(C_k \cap A)}{P(A)}$$

$$= \frac{P(C_k)\, P(A | C_k)}{\sum_{i=1}^{\infty} P(C_i)\, P(A | C_i)}$$

<u>Bayes' Rule</u>

<u>Ex</u>: Let A: person uses substance ②

B: test result is positive

Assume: 1 in 20 persons use the substance

$P(A) = .05$

If the person uses the substance,
the test results will be positive
99% of the time.

$P(B | A) = .99$

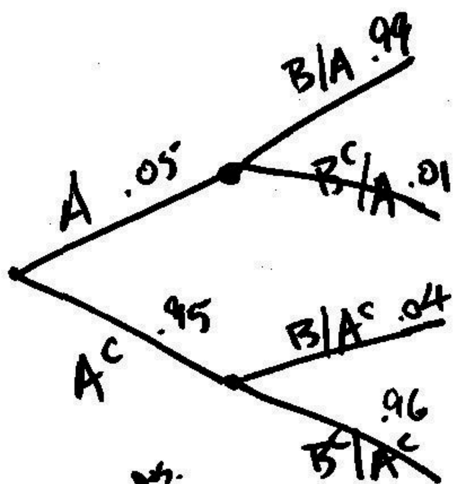
③

If the person does not use the substance, the test results will be negative 96% of the time.

$$P(B^c \mid A^c) = .96$$

Question: Given that the test results are positive, what is the probability that the person uses the substance?

④



$.0495 = P(A)\,P(B|A) = P(A \cap B)$

$.0005 = P(A \cap B^c)$

$.038 = P(A^c \cap B)$

$.912 = P(A^c \cap B^c)$

true pos.

false neg.

true neg.

false pos.

	B	B^c	
A	.0495	.0005	.0500
A^c	.0380	.9120	.9500
	.0875	.9125	1

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.0495}{.0875} = .566 \text{ !!!}$$

(5)

How does this use Bayes' Rule?

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)}$$

$$= \frac{(.05)(.99)}{(.05)(.99) + (.95)(.04)} = .566$$

(6)

Assume $P(M) = P(F) = .5$

Assume independent births

You see a person with one of their children, a son.

You know that they have 2 children.

Find the prob. that their other child is a son.

Let A: 1^{st} child is M

B: 2^{nd} child is M

Find $P(A \cap B | A \cup B) = \dfrac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)}$

$$= \frac{P(A \cap B)}{P(A \cup B)}$$

	P
MM	$1/4$
MF	$1/4$
FM	$1/4$
FF	$1/4$

$$= \frac{1/4}{3/4} = \frac{1}{3}$$

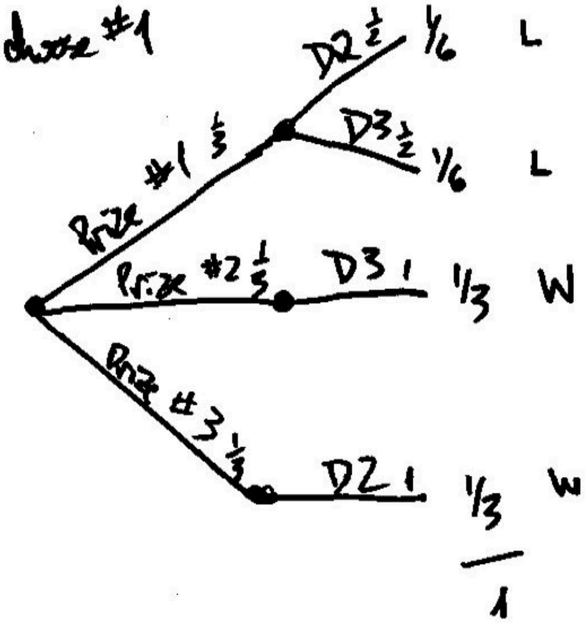
<u>Monty Hall problem</u>

You choose door #1

Monty Hall opens one of the 2 remaining doors to reveal a joke prize. He then gives you the chance to switch your choice. Should you?

Assume that you switch.

You choose #1



Switch $\Rightarrow$ $P(W) = 2/3$

(9)

Random variables

Defn: A random variable is a function from the sample space into the set of real numbers.

$$X : S \to \mathbb{R}$$

Defn: The range of the random variable X is the set of real numbers that X maps onto.

(10)

Example: Flip 2 coins

$$S = \{HH, HT, TH, TT\}$$

Let X be the # of heads

$$X(HH) = 2$$
$$X(HT) = 1$$
$$X(TH) = 1$$
$$X(TT) = 0$$

range of X is $\{0,1,2\}$

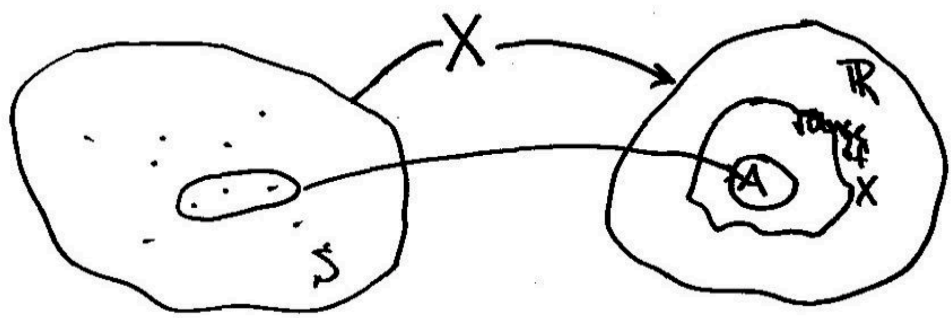
Defn: If the range of X is countable (finite or countably infinite), then X is a discrete random variable.

(11)

If the range of X is an interval, then X is a continuous random variable.

Defn: Let A be a subset of the range of X.

Define $P_X(A) = P\left[\{s \in S : X(s) \in A\}\right]$



(12)

Example: Roll 2 dice

$$S = \left\{ \begin{matrix} 11 & 12 & \cdots & 16 \\ 21 & 22 & \cdots & 26 \\ \vdots & & & \\ 61 & 62 & \cdots & 66 \end{matrix} \right\}$$

Let X be the maximum of the 2 dice.

Find the probability that X = 5.

$$P_X(\{5\}) = P\left[\{15, 25, 35, 45, 51, 52, 53, 54, 55\}\right]$$

$$= \frac{9}{36} = \frac{1}{4}$$

Consider the discrete case (13)

Defn: Let $r_i \in$ range of X

Let $p_X(r_i) = P_X(\{r_i\})$.

p_X is called a probability mass function (pmf)

Defn: Let $F_X(x) = P_X[(-\infty, x]]$

This is called the cumulative distribution function (cdf) of the random variable X.

(14)

Example: Flip 3 coins.

Let $X = $ # heads.

Find the pmf & cdf for X.

$$S = \{HHH, HTH, HHT, HTT, THH, THT, TTH, TTT\}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$3 \quad 2 \quad 2 \quad 1 \quad 2 \quad 1 \quad 1 \quad 0$$

pmf: $p_X(0) = \frac{1}{8}$

$p_X(1) = 3/8$

$p_X(2) = 3/8$ $\qquad p_X(1.5) = 0$

$p_X(3) = \frac{1}{8}$

cdf: $F(x) = P(X \le x)$