451

(1)

5-17

Poisson Distribution

(1) Observe a process over a fixed amount of time or space

(2) X counts the number of occurrences of a certain type

(3) Occurrences in non-overlapping intervals are independent

(4) Prob. of an occurrence in an interval is proportional to the length of the interval

(5) In a very small interval, the prob. of more than 1 occurrence ≈ 0

$$p(x) = \frac{\mu^x e^{-\mu}}{x!} \quad x = 0, 1, 2, \ldots$$

(2)

Check to see if this is valid:

$$\sum_{x=0}^{\infty} \frac{\mu^x e^{-\mu}}{x!} = e^{-\mu} \sum_{x=0}^{\infty} \frac{\mu^x}{x!}$$

$$= e^{-\mu}\left[1 + \mu + \frac{\mu^2}{2!} + \frac{\mu^3}{3!} + \ldots\right]$$

$$= e^{-\mu} \cdot e^{\mu} = 1$$

Let $\mu = \lambda t$

$$p(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

③

<u>Example</u>: Watch the bank entrance for 20 minutes & count customers.

On the average, there are 30 customers per hour.

Poisson$(\mu = \lambda t = 30 \cdot \frac{1}{3} = 10)$

Find the probability of seeing exactly 5 customers.

$$P(X=5) = p(5) = \frac{10^5 e^{-10}}{5!} = .0378$$

<u>Example</u>: A paramedic squad gets 2.83 calls, on the average, per day.

Find the prob. that they get at most 4 calls in a day.

④

Poisson$(\mu = 2.83)$

$$P(X \le 4) = p(0) + p(1) + p(2) + p(3) + p(4)$$

$$= \frac{2.83^0 e^{-2.83}}{0!} + \cdots + \frac{2.83^4 e^{-2.83}}{4!}$$

$$= .8429$$

In this same example, find $P(X > 7)$

$p(8) + p(9) + \cdots$

or $P(X>7) = 1 - P(X \le 7)$

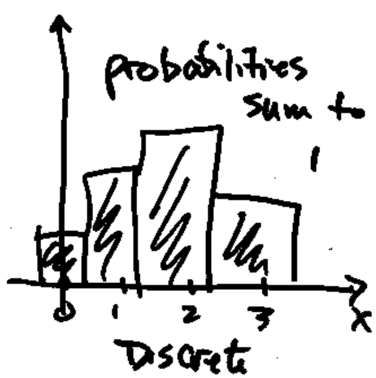
Mean of Poisson is μ

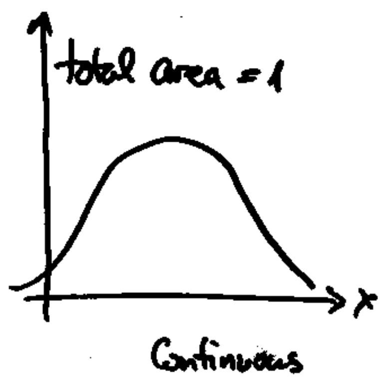
Variance of binomial was npq

$$= n\left(\frac{\mu}{n}\right)\left(1 - \frac{\mu}{n}\right) \xrightarrow[n\to\infty]{} \mu$$

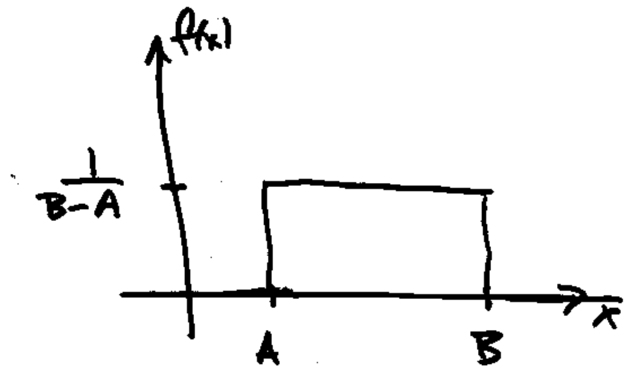
Discrete Distributions (Summary)

- Discrete uniform
- Bernoulli
- Binomial
- Geometric
- Negative binomial (Pascal)
- Hypergeometric
- Poisson





Continuous Uniform distribution



$$\boxed{f(x) = \begin{cases} \dfrac{1}{B-A} & A < x < B \\ 0 & \text{elsewhere} \end{cases}}$$

$$\mu = \frac{A+B}{2}$$

Find σ^2:

$$E[X^2] = \int_A^B x^2 \frac{1}{B-A}\, dx$$

$$= \frac{1}{B-A}\ \frac{x^3}{3}\Big|_A^B$$

$$= \frac{1}{B-A}\cdot\frac{1}{3}\cdot(B^3-A^3)$$

$$= \frac{1}{3}(B^2+AB+A^2)$$

$$\sigma^2 = E[X^2]-\mu^2$$

$$= \frac{1}{3}(B^2+AB+A^2) - \left(\frac{A+B}{2}\right)^2$$

$$= \frac{1}{12}\left[4B^2+4AB+4A^2-(3A^2+6AB+3B^2)\right]$$

$$= \frac{1}{12}\left[B^2-2AB+A^2\right]$$

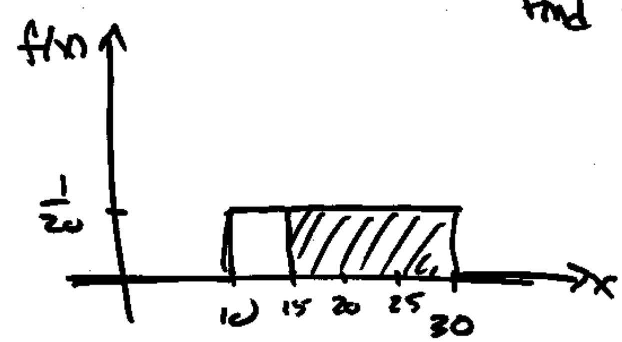
$$= \frac{(B-A)^2}{12}$$

Example: Travel time to work is uniformly distributed between 10 minutes and 30 minutes.

$$\text{Unif}(A=10,\ B=30)$$

$$f(x) = \begin{cases} \frac{1}{20} & 10<x<30 \\ 0 & \text{elsewhere} \end{cases}$$

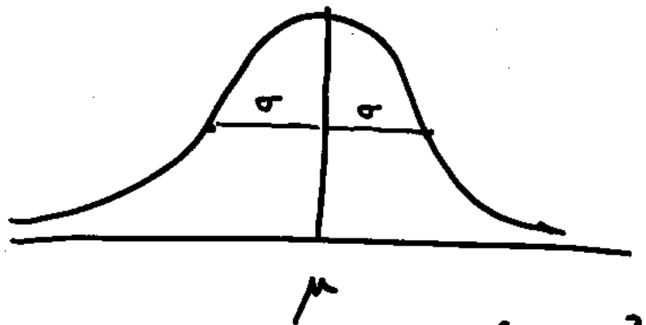
Find $P(X>15)$



$$\text{Area} = 15\cdot\frac{1}{20} = .75$$

(9)

The normal distribution



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

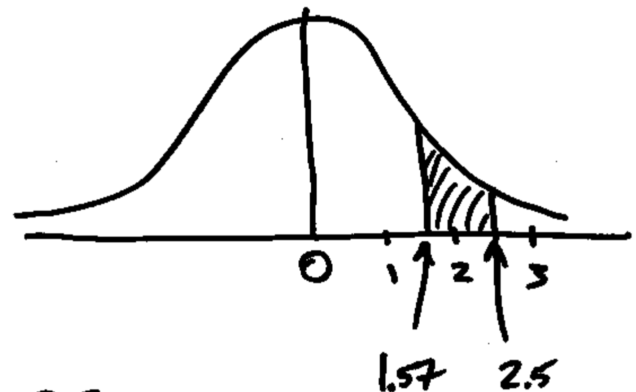
$$-\infty < x < \infty$$

Standard normal distribution ($\mu = 0, \sigma = 1$)

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

(10)

Example: In the standard normal distribution find the prob. of seeing a value between 1.57 and 2.5



$$\int_{1.57}^{2.5} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx \leftarrow \text{solve numerically}$$

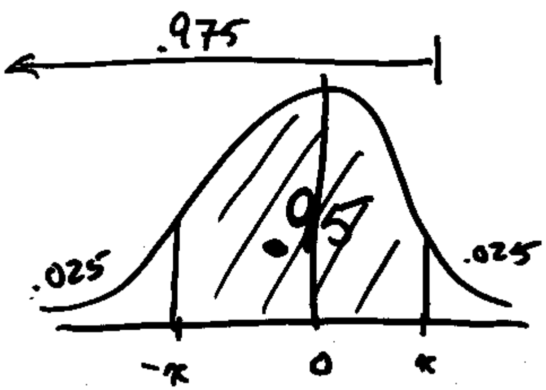
Using the table, find $F(2.5) - F(1.57)$
$= .9938 - .9418 = .052$

(11)

Ex: For the standard normal distr.,

find the x values such that

$$P(-x < X < x) = .95$$



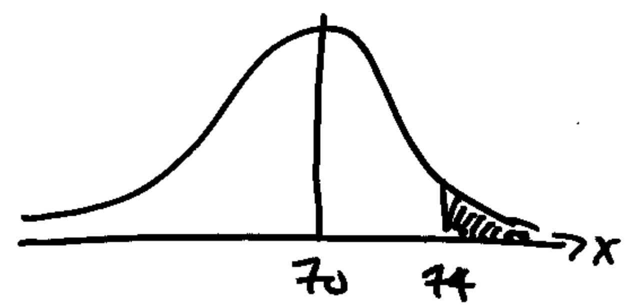
$x = 1.96$

(12)

Ex: A population of measurements is normally distributed with $\mu = 70$ mm and $\sigma = 2$mm

What percentage of the items are longer than 74 mm?

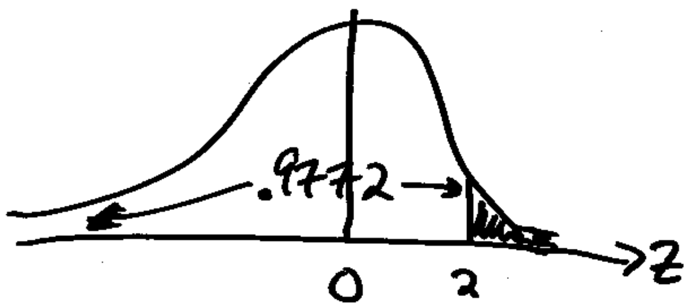
Normal($\mu = 70, \sigma = 2$) $\quad P(X > 74)$



Transformation: $\quad Z = \dfrac{X - \mu}{\sigma}$
(z-score)

After transformation,

$$\text{Normal}(\mu=0, \sigma=1) \quad P(Z > 2)$$



$$1 - .9772 = .0228 = 2.28\%$$

HW#7 p.165 # 52, 58, 76

p.186 # 4, 10, 12, 22

13

Stat 451 HW#6 p.150 # 2,12,16
p.157 # 30,32,48

2pts each
12 pts total

2. Bino$(n=12, p=.5)$

$$P(X=3) = \binom{12}{3}(.5)^3(.5)^9 = \underline{.0537}$$

12. Bino$(n=9, p=.75)$ OR Bino$(n=9, p=.25)$

$P(X>5) = P(X \geq 6)$

$= p(6) + p(7) + p(8) + p(9)$

$P(X<4) = P(X \leq 3)$

$= p(0) + p(1) + p(2) + p(3)$

$= \underline{.8343}$

16. 4-engine plane

Bino$(4, .4)$ fail OR Bino$(4, .6)$ don't fail

$P(X \leq 2)$ | $P(X \geq 2)$

$= .8208$

2-engine plane

Bino$(2, .4)$ fail OR Bino$(2, .6)$ don't fail

$P(X \leq 1)$ | $P(X \geq 1)$

$= .84$

So, the 2-engine plane has a higher probability of success.

30. Hyper($N=15, k=6, n=3$)

$P(X \geq 1) = 1 - p(0)$

$= 1 - \dfrac{\binom{6}{0}\binom{9}{3}}{\binom{15}{3}}$

$= 1 - .1846$

$= \underline{.8154}$

32. Hyper($N=10, k=3, n=4$)

a) $P(X=0) = \dfrac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} = \underline{.1667}$

b) $P(X \leq 2) = p(0) + p(1) + p(2)$

$= .1667 + \dfrac{\binom{3}{1}\binom{7}{3}}{\binom{10}{4}} + \dfrac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}}$

$= .1667 + .5 + .3 = \underline{.9667}$

48. Hyper($N=15, k=2, n=5$)

a) $P(X=1) = \dfrac{\binom{2}{1}\binom{13}{4}}{\binom{15}{5}} = \underline{.4762}$

b) $P(X=2) = \binom{2}{2}\binom{13}{3}/\binom{15}{5} = \underline{.0952}$