Chapter 5 451

① 5-K)

Discrete Distributions

Discrete uniform distribution

- X takes on values $x_1, x_2, \dots, x_k$, each with probability $\frac{1}{k}$.

$$\left\{ p(x) = \frac{1}{k} \qquad x = x_1, \dots, x_k \right\}$$

Special case: X takes on values $1, 2, \dots, k$

Find μ and σ^2

$$\mu = E[X] = \sum_{x=1}^{k} x\, p(x)$$

②

$$= \sum_{x=1}^{k} x \cdot \frac{1}{k} = \frac{1}{k} \sum_{x=1}^{k} x$$

$$= \frac{1}{k} \frac{k(k+1)}{2} = \frac{k+1}{2}$$

$$E[X^2] = \sum_{x=1}^{k} x^2 p(x) = \sum_{x=1}^{k} x^2 \cdot \frac{1}{k}$$

$$= \frac{1}{k} \sum_{x=1}^{k} x^2$$

$$= \frac{1}{k} \frac{k(k+1)(2k+1)}{6} = \frac{(k+1)(2k+1)}{6}$$

$$\sigma^2 = E[X^2] - (E[X])^2$$

$$= \frac{(k+1)(2k+1)}{6} - \frac{(k+1)^2}{4}$$

$$= (k+1) \frac{(2k+1)2 - (k+1)3}{12}$$

③

$$= (k+1)\frac{4k+2-3k-3}{12}$$

$$= \frac{(k+1)(k-1)}{12} = \frac{k^2-1}{12}$$

Bernoulli Distribution

$$X = \begin{cases} 1 & p \\ 0 & 1-p = q \end{cases}$$

Find μ and σ^2

$$E[X] = \sum_x x\,p(x) = 1\cdot p + 0(1-p)$$
$$= p$$

$$E[X^2] = \sum x^2 p(x) = 1^2\cdot p + 0^2(1-p)$$
$$= p$$

④

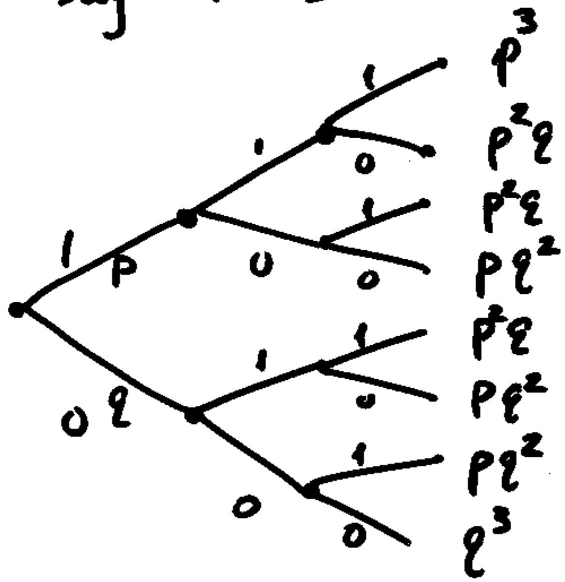
$$\sigma^2 = p - p^2 = p(1-p)$$
$$= pq$$

Binomial Distribution

- Run a sequence of n identical Bernoulli trials
- Assume independence of trials
- X counts the number of "1"s in the Bernoulli trials.

$$\boxed{\begin{gathered} X = 0, 1, \ldots, n \\ p(x) = \binom{n}{x} p^x q^{n-x} \end{gathered}}$$

Say $n = 3$



x	$p(x)$
0	q^3
1	$3pq^2$
2	$3p^2q$
3	p^3

$$p(x) = \binom{3}{x} p^x q^{3-x}$$

Find μ, σ^2.

Let $X_1, X_2, \ldots, X_n$ be the Bernoulli random variables

Know $E[X_i] = p$

and $V[X_i] = pq$ $\forall i$

$$X = X_1 + X_2 + \ldots + X_n$$

$$E[X] = E[X_1] + \ldots + E[X_n] = np$$

$$V[X] = V[X_1] + \ldots + V[X_n] = npq$$

Example: Aim a projectile at a target & shoot 20 times. 70% of the shots hit the target.

Find the expected number of hits, the standard deviation of the number of hits and the probability of hitting exactly 15 out of 20.

$\underline{\text{Binomial}}\,(\underbrace{n = 20,\ p = .7}_{\text{parameters}})$

a) $E[X] = \mu = np = 20(.7) = 14$

b) $\sigma = \sqrt{\sigma^2} = \sqrt{npq} = \sqrt{20(.7)(.3)}$

$= \sqrt{4.2} = 2.049$

c) $P(X=15) = p(15)$

$= \binom{20}{15}(.7)^{15}(.3)^5 = .1789$

Find the prob. that X is greater than 15. $P(X>15) = p(16) + \ldots + p(20)$.

Hypergeometric Distribution

- Population consisting of N items
- K of those items have a particular characteristic.
- Take a sample of n items, WOR
- X counts the number of items in the sample, having that characteristic.

⑨

$$X = 0, 1, \ldots, \min(n, k)$$

$$p(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

Example From a pool of 120 females and 80 males, we randomly select a jury of 12.

Find the probability that the jury contains exactly 6 females.

Hyper$(N = 200, k = 120, n = 12)$

⑩

$$P(X = 6) = p(6)$$

$$= \frac{\binom{120}{6}\binom{80}{6}}{\binom{200}{12}} = .18$$

$$\mu = E[X] = n \frac{k}{N}$$

$$\sigma^2 = n \frac{k}{N}\left(1 - \frac{k}{N}\right)\left(\frac{N-n}{N-1}\right)$$

HW #6 due Thursday 5/17

p. 150 # 2, 12, 16

p. 157 # 30, 32, 48

STAT 451 HW#5 p.134 # 58, 62, 70, 72 2pts each

8 pts total

58.

$f(x,y)$	$x=2$	$x=4$	
$y=1$	.1	.15	.25
$y=3$	.2	.3	.5
$y=5$	.1	.15	.25
	.4	.6	

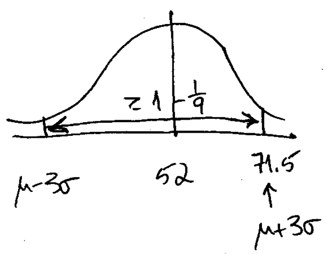
$E(X) = 2(.4) + 4(.6)$
$= .8 + 2.4 = 3.2$

$E(Y) = 1(.25) + 3(.5) + 5(.25)$
$= .25 + 1.5 + 1.25 = 3$

a) $E(2X-3Y) = 2E(X) - 3E(Y)$
$= 2(3.2) - 3(3) = 6.4 - 9 = -2.6$

b) $E(XY) = (1)(2)(.1) + (1)(4)(.15) + (3)(2)(.2) + (3)(4)(.3)$
$+ (5)(2)(.1) + 5(4)(.15)$
$= .2 + .6 + 1.2 + 3.6 + 1 + 3 = 9.6$

62. $\mu = 52$, $\sigma = 6.5$, symmetric distr.



Tails, together $< 1/9$

So $\boxed{P(X > 71.5) < 1/18}$

70. X,Y indep. $g(x) = 8x^{-3}$, $x > 2$

$Z = XY$ $h(y) = 2y$, $0 < y < 1$

Find $E(Z)$.

By indep., $E(Z) = E(X)E(Y)$.

$E(X) = \int_2^\infty 8x^{-2}\,dx = \left.\frac{8x^{-1}}{-1}\right|_2^\infty = 0 + 4 = 4$

$E(Y) = \int_0^1 2y^2\,dy = \left.\frac{2y^3}{3}\right|_0^1 = 2/3$

So $\boxed{E(Z) = 8/3}$

72. X = number on 1st die Y = number on 2nd die.

So $E(X) = E(Y) = 1(\frac{1}{6}) + \ldots + 6(\frac{1}{6}) = 3.5$
$E(X^2) = E(Y^2) = 1^2(\frac{1}{6}) + \ldots + 6^2(\frac{1}{6}) = \frac{91}{6}$
$Var(X) = Var(Y) = \frac{91}{6} - (\frac{7}{2})^2 = \frac{35}{12}$

a) Find $V(2X - Y) = 4\,Var X + Var Y = 5 \cdot \frac{35}{12} = \frac{175}{12}$

b) Find $V(X + 3Y - 5) = Var X + 9\,Var Y = 10 \cdot \frac{35}{12} = \frac{175}{6}$