Chebyshev's Inequality

451

1

5-3

$$\sigma^2 = E[(X-\mu)^2]$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

$$= \underbrace{\int_{-\infty}^{\mu-k\sigma} (x-\mu)^2 f(x)\,dx}_{1} + \underbrace{\int_{\mu-k\sigma}^{\mu+k\sigma} (x-\mu)^2 f(x)\,dx}_{2}$$

$$+ \underbrace{\int_{\mu+k\sigma}^{\infty} (x-\mu)^2 f(x)\,dx}_{3}$$

$$\geq 1 + 3$$

2

Look at 1.

Everywhere in the range of integration, $X < \mu - k\sigma$

$$X - \mu < -k\sigma$$

$$(X-\mu)^2 > k^2\sigma^2$$

Look at 3.

Everywhere in the range of integration, $X > \mu + k\sigma$

$$X - \mu > k\sigma$$

$$(X-\mu)^2 > k^2\sigma^2$$

Now $\sigma^2 \geq 1 + 3$

$$> \int_{-\infty}^{\mu-k\sigma} k^2\sigma^2 f(x)\,dx + \int_{\mu+k\sigma}^{\infty} k^2\sigma^2 f(x)\,dx$$

3

$$= k^2\sigma^2\left[\int_{-\infty}^{\mu-k\sigma} f(x)\,dx + \int_{\mu+k\sigma}^{\infty} f(x)\,dx\right]$$

$$= k^2\sigma^2\left[P(X<\mu-k\sigma) + P(X>\mu+k\sigma)\right]$$

$$= k^2\sigma^2\left[P(X-\mu<-k\sigma) + P(X-\mu>k\sigma)\right]$$

$$= k^2\sigma^2\left[P(|X-\mu|>k\sigma)\right]$$

$$\sigma^2 > k^2\sigma^2 P(|X-\mu|>k\sigma)$$

$$\boxed{P(|X-\mu|>k\sigma) < \frac{1}{k^2}}$$

$$\boxed{P(|X-\mu|<k\sigma) \geq 1-\frac{1}{k^2}}$$

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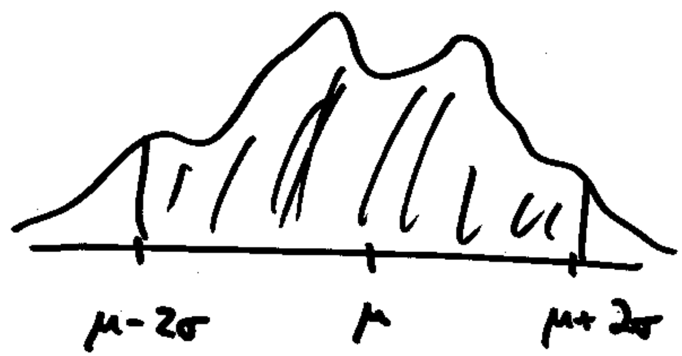
Example: For any continuous distribution, how much of the probability lies in the range from $\mu-2\sigma$ to $\mu+2\sigma$?



At least $1-\frac{1}{2^2} = 75\%$

⑤

Midterm exam Tuesday May 8

1 page of notes ($8\frac{1}{2} \times 11$, 2-sided)

Calculator

- Descriptive Statistics

  Measures of location and dispersion

- Counting Rules (6)

- Unions, Intersections, Complements

- Conditional probability & Bayes' Rule

- Random variables & probability distributions

- Cumulative distribution

⑥

- $E[X]$, $E[X^2]$, $V(X)$, σ

- joint distributions

- $E[XY]$, $Cov(X,Y)$, ρ

- Independence

- Chebyshev's Rule