Stat 451 Hw #9 p.226 #18, p.252 #24, 26

18. $M_x(t) = E[e^{tx}] = \sum_{x=1}^{\infty} e^{tx} p q^{x-1}$

$= e^t p \sum_{x=1}^{\infty} (qe^t)^{x-1} = \frac{pe^t}{1-qe^t} \underbrace{\sum_{x=1}^{\infty} (1-qe^t)(qe^t)^{x-1}}$

Geometric with $p^* = 1-qe^t$, $q^* = qe^t$

$= \frac{pe^t}{1-qe^t}$

Then $M_x'(t) = \frac{(1-qe^t)pe^t - pe^t(-qe^t)}{(1-qe^t)^2}$

$= pe^t(1-qe^t)^{-2}$

$\mu = M_x'(0) = p \cdot p^{-2} = \boxed{\frac{1}{p}}$

And $M_x''(t) = \frac{(1-qe^t)^2 pe^t - pe^t 2(1-qe^t)(-qe^t)}{(1-qe^t)^4}$

$E(x^2) = M_x''(0) = \frac{p^3 + 2p^2 q}{p^4} = \frac{1}{p} + \frac{2q}{p^2}$

$\sigma^2 = \frac{1}{p} + \frac{2q}{p^2} - \frac{1}{p^2} = \frac{1}{p} \frac{p+2q-1}{p^2} = \boxed{\frac{q}{p^2}}$

24. $\mu = 40$, $\sigma = 2$, $n = 30$

$P\left(\sum_{i=1}^{30} x_i > 1458\right) = P(\bar{x} > 48.6)$

$= P\left(z > \frac{48.6-40}{2/\sqrt{30}}\right) = P(z > 23.55)$

$\approx \underline{0}$

26. $\mu = 3.2$, $\sigma = 1.6$, $n = 64$

a) $P(\bar{x} \le 2.7) = P\left(z \le \frac{2.7-3.2}{1.6/\sqrt{64}}\right) = P(z \le -2.5)$

$= \underline{.0062}$

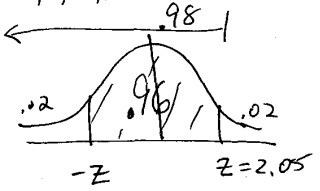
b) $P(\bar{x} > 3.5) = P(z > 1.5) = 1 - .9332 = \underline{.0668}$

c) $P(3.2 \le \bar{x} < 3.4) = P(0 < z < 1) = .8413 - .5$

$= \underline{.3413}$

Stat 451 Practice problems p. 285 # 4, 8, 14, 16

4. $\sigma = 40, n = 30, \bar{x} = 780$



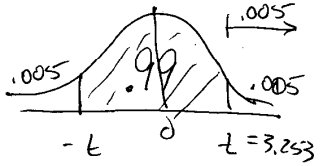
96% C.I. for μ: $\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

$780 \pm (2.05)\frac{40}{\sqrt{30}}$

780 ± 14.97

8. $n = \left(\frac{z\sigma}{e}\right)^2 = \left\lfloor\frac{2.05(40)}{10}\right\rfloor^2 = 68$

14. $n = 10, \bar{x} = 230, s = 15$

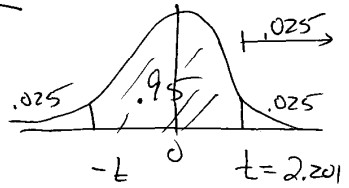


99% C.I. for μ: $\bar{x} \pm t\frac{s}{\sqrt{n}}$

$230 \pm (3.253)\frac{15}{\sqrt{10}}$ $df = 9$

230 ± 15.42

16. $n = 12, \bar{x} = 79.3, s = 7.8$



95% C.I. for μ: $\bar{x} \pm t\frac{s}{\sqrt{n}}$ $df = 11$

$79.3 \pm (2.201)\frac{7.8}{\sqrt{12}}$

79.3 ± 4.96