p.89 451

3.24 5 jazz 1

2 classical 4-24

3 rock

10 CDs

Select 4 at random.

$X = \#$ jazz CDs

X	$p(X)$
0	$\binom{5}{4}/\binom{10}{4}$
1	$\binom{5}{1}\binom{5}{3}/\binom{10}{4}$
2	$\binom{5}{2}\binom{5}{2}/\binom{10}{4}$
3	$\binom{5}{3}\binom{5}{1}/\binom{10}{4}$
4	$\binom{5}{4}/\binom{10}{4}$

$$p(x) = \frac{\binom{5}{x}\binom{5}{4-x}}{\binom{10}{4}}$$

$X = 0, 1, \ldots, 4$

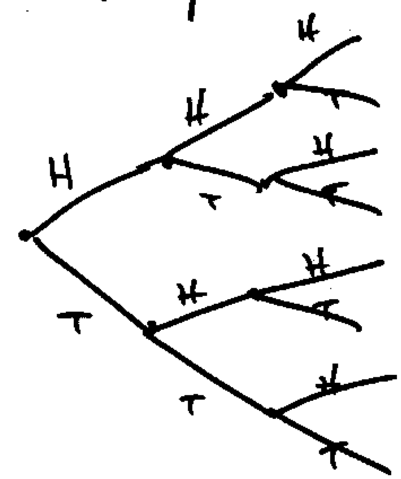
p.88 2

3.8 3 coin tosses

$W = \#\text{heads} - \#\text{tails}$

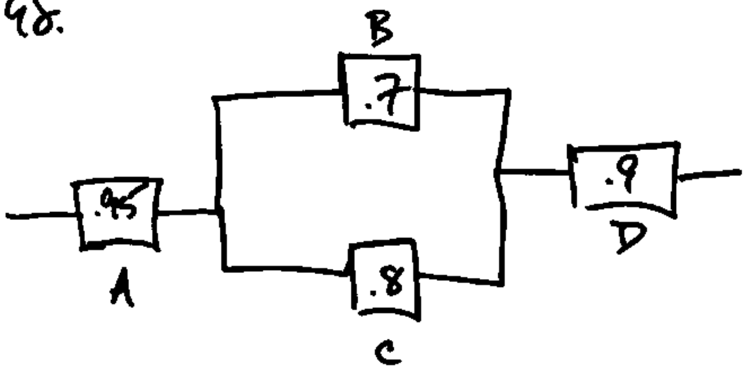
$P(H) = 2/3$

# heads	W	$p(w)$
0	-3	$(1/3)^3$
1	-1	$3(2/3)(\frac{1}{3})^2$
2	1	$3(2/3)^2(\frac{1}{3})^1$
3	3	$(\frac{2}{3})^3$



p.67 (3)

98.



$$P(A \cap (B \cup C) \cap D)$$

$$= P(A)\,P(B \cup C)\,P(D) \quad \text{by independence}$$

$$= (.95)[.94](.9) = .8037$$

$$\left(\begin{aligned} P(B \cup C) &= P(B) + P(C) - P(B \cap C) \\ &= .7 + .8 - (.7)(.8) = .94 \end{aligned} \right)$$

p.66 (4)

86. $P(H) = .21$

$P(W) = .28$

$P(H \cap W) = .15$

a) $P(H \cup W) = .21 + .28 - .15 = .34$

b) $P(W|H) = \dfrac{.15}{.21} = .71$

c) $P(H|W') = \dfrac{P(H \cap W')}{P(W')}$

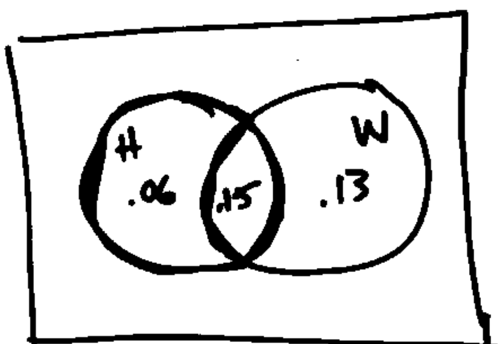
$$\begin{aligned} P(H \cap W') &= P(H)\,P(W'|H) \\ &= .21\,[1 - .71] = .21(.29) \end{aligned}$$

c) $= \dfrac{.21(.29)}{.72} = .08$

5

	W	W'	
H	.15	.06	.21
H'	.13	.66	.79
	.28	.72	1

c) $P(H|W') = \frac{.06}{.72} = .08$



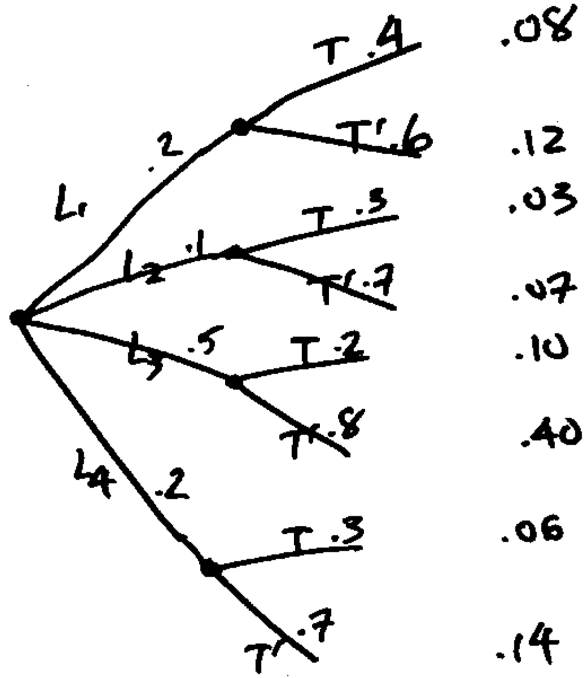
$P(H \cap W') = .06$

$H = (H \cap W') \cup (H \cap W)$

6

p.72

102.



	T	T'	
L_1	.08	.12	.2
L_2	.03	.07	.1
L_3	.10	.40	.5
L_4	.06	.14	.2
	.27	.73	1

$P(T) = .27$

104. $P(L_2|T)$

$= \frac{.03}{.27}$

$= .11$

7

Joint density function for
2 continuous random variables

Example: $f(x,y) = \begin{cases} 8xy & 0 \le y \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$

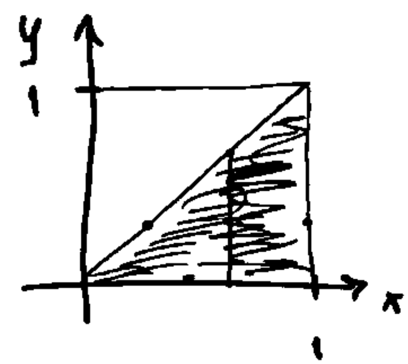
Probabilities are found by integration.

If A represents a region in the x-y plane, then

$$P(A) = P((x,y) \in A) = \iint_A f(x,y)\,dx\,dy$$

8

Is our example a valid density?



$0 \le y \le x \le 1$

$$\int_0^1 \int_y^1 8xy \, dx\, dy$$

$$= \int_0^1 8y \frac{x^2}{2}\Big|_y^1 \, dy$$

$$= \int_0^1 4y(1-y^2)\, dy$$

$$= (2y^2 - y^4)\Big|_0^1 = 1$$

OR

$$\int_0^1 \int_0^x 8xy \, dy\, dx$$

$$= \int_0^1 8x \frac{y^2}{2}\Big|_0^x \, dx$$

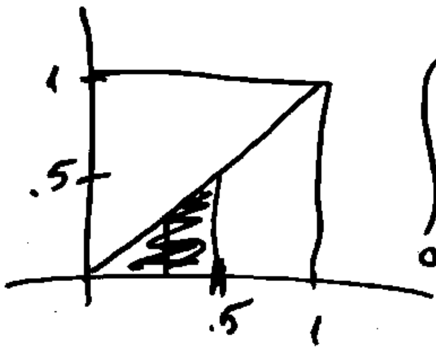
$$= \int_0^1 4x^3 \, dx$$

$$= x^4\Big|_0^1 = 1$$

(9)

Find the probability that

X and Y are both less than .5



$$\int_0^{.5}\int_0^{x} 8xy \, dy\, dx$$

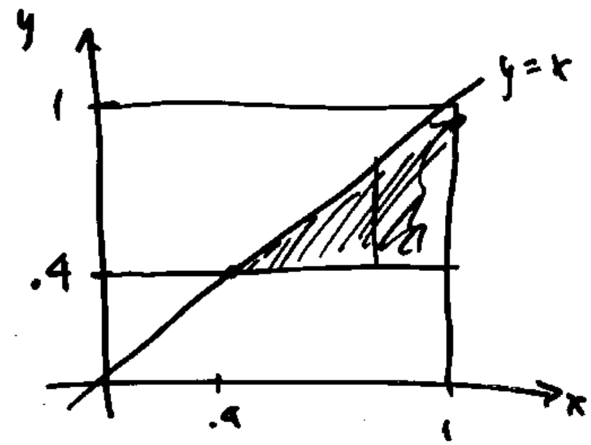
$$= \int_0^{.5} 8x \frac{y^2}{2}\Big|_0^{x} \, dx$$

$$= \int_0^{.5} 4x^3\, dx = x^4\Big|_0^{.5} = \frac{1}{16}$$

(10)

Set up only:

Find the probability that $y > .4$



$$\int_{.4}^{1}\int_{.4}^{x} 8xy \, dy\, dx$$

(11)

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

Example $f(x,y) = 8xy \quad 0 \le y \le x \le 1$

Find $g(x) = \int_0^x 8xy\,dy$

$$= 8x \frac{y^2}{2}\Big|_0^x = 4x^3$$

$$g(x) = \begin{cases} 4x^3 & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

(12)

Find $h(y) = \int_y^1 8xy\,dx$

$$= 8y \frac{x^2}{2}\Big|_y^1$$

$$h(y) = \begin{cases} 4y(1-y^2) & 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

Definition: X and Y are independent if $f(x,y) = g(x)h(y) \quad \forall x,y$

In our example, are X & Y independent?

$$8xy \stackrel{?}{=} 4x^3 \cdot 4y(1-y^2) \qquad \text{No.}$$

13

HW #4 due 5/1

p. 101 #40, 44, 52

p. 113 # 10, 12, 26

p. 122 # 46, 50