p.48

451

1

4-17

48. Cars hold 2, 4, 5

9 people:

2	4	3
2	3	4
2	2	5
1	4	4
1	3	5

$$\frac{9!}{2!\,4!\,3!} + \frac{9!}{2!\,3!\,4!} + \frac{9!}{2!\,2!\,5!} + \frac{9!}{1!\,4!\,4!} + \frac{9!}{1!\,3!\,5!}$$

$$= 1260 + 1260 + 756 + 630 + 504 = 4410$$

2

50. 365 possible birthdays

60 people

P(no birthdays in common)

$$= \frac{n(\text{no birthdays in common})}{n(S)}$$

$$= \frac{365 \cdot 364 \cdot \ldots \cdot (365-60+1)}{365^{60}}$$

$$= \frac{P^{365}_{60}}{365^{60}}$$

= PERMUT(365,60)/365^60

in Excel

= .0059

③

p.56

64. Roll 5 dice

P(4 of a kind)

$$= \frac{\underset{\leftarrow \text{\# ways of selecting the repeated die}}{6} \cdot \underset{\leftarrow \text{\# ways of selecting the remaining die}}{5} \cdot \underset{\leftarrow \text{\# of different positions for the "odd" die}}{5}}{6^5}$$

$= .0193$ $\quad \left(+ \frac{6}{6^5} \right.$ if you include 5 of a kind $\left. \right)$

58. a) $P(B \cup F) = P(B) + P(F) - P(B \cap F)$

$= .25 + .17 - .15$

$= .27$

b) $P(B \cup F)' = 1 - P(B \cup F)$

$= 1 - .27 = .73$

④

p.48

38. a) $8!$

b) $4! \cdot 2^4$

c) $(4!)^2$

Cumulative distribution function

$$F(x) = \sum_{t \le x} p(t)$$

Find the prob. that $x = 5$: $\quad p(5)$

Find the prob that $x \le 5$: $\quad F(5)$

Ex: Flip 3 coins. X = # heads

$F(2) = P(X \le 2) = p(2) + p(1) + p(0)$

Continuous random variables

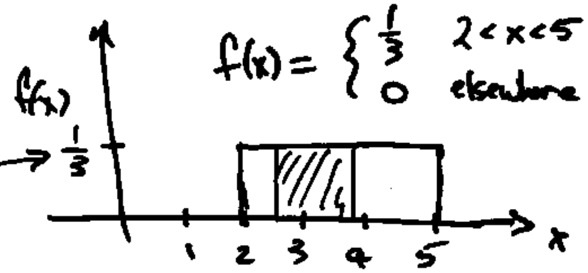
$$P(a < X < b) = \int_a^b \underbrace{f(x)}\,dx$$

probability density function

Example, say that X takes on all possible values between 2 and 5, with equal likelihood throughout the interval

$$f(x) = \begin{cases} \frac{1}{3} & 2 < x < 5 \\ 0 & \text{elsewhere} \end{cases}$$

Forces the total area to be 1



$$P(2.5 < X < 3.7) = \int_{2.5}^{3.7} f(x)\,dx$$

$$= \int_{2.5}^{3.7} \tfrac{1}{3}\,dx = \tfrac{1}{3}x \Big|_{2.5}^{3.7}$$

$$= \tfrac{1}{3}(3.7 - 2.5)$$

$$= \tfrac{1}{3}(1.2) = .4$$

Same setup,

$$P(1 < X < 4) = \int_1^4 f(x)\,dx$$

$$= \underbrace{\int_1^2 f(x)\,dx}_{=\,0} + \int_2^4 f(x)\,dx$$

$$= \int_2^4 \tfrac{1}{3}\,dx = \tfrac{2}{3}$$

7

Cumulative distribution function

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

$$= P(X \leq x)$$

$$P(a < X < b) = \int_{a}^{b} f(x)\,dx$$

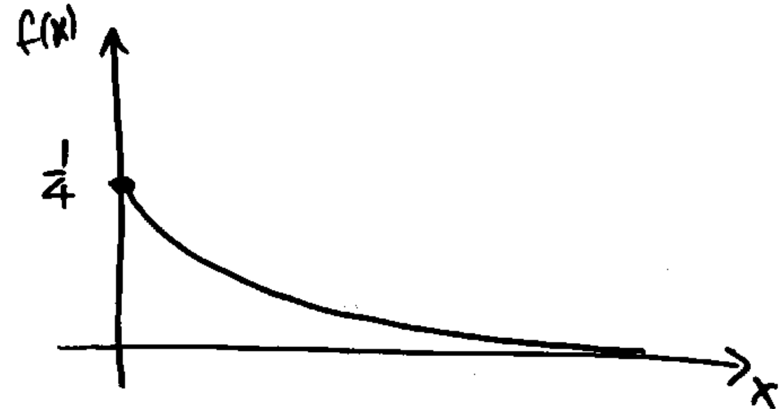
$$= \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx$$

$$= F(b) - F(a)$$

$$f(x) = \frac{d}{dx} F(x)$$

8

Example : $f(x) = \begin{cases} \frac{1}{4} e^{-\frac{1}{4}x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$



Check that this a valid probability density:

$$P(S) = \int_{0}^{\infty} \frac{1}{4} e^{-\frac{1}{4}x}\,dx = \frac{1}{4} \frac{e^{-\frac{1}{4}x}}{-\frac{1}{4}} \Bigg|_{0}^{\infty}$$

$$= -e^{-\frac{1}{4}x} \Big|_{0}^{\infty}$$

$$= 0 - (-1) = 1 \quad \checkmark$$

(9)

Find $P(1 < X < 2)$

$$= \int_1^2 \frac{1}{4} e^{-\frac{1}{4}x} dx$$

$$= \frac{1}{4} \frac{e^{-\frac{1}{4}x}}{-\frac{1}{4}} \Big|_1^2$$

$$= -e^{-\frac{1}{4}x} \Big|_1^2$$

$$= -e^{-\frac{1}{2}} - (-e^{-\frac{1}{4}})$$

$$= e^{-\frac{1}{4}} - e^{-1/2}$$

Find the c.d.f., $F(x)$

(10)

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

$$= \int_{-\infty}^{0} \underset{0}{f(t)}dt + \int_0^x f(t)dt$$

$$= \int_0^x \frac{1}{4} e^{-\frac{1}{4}t} dt$$

$$= \frac{1}{4} \frac{e^{-\frac{1}{4}t}}{-\frac{1}{4}} \Big|_0^x$$

$$= -e^{-\frac{1}{4}t} \Big|_0^x$$

$$= -e^{-\frac{x}{4}} - (-1)$$

$$= 1 - e^{-x/4}$$

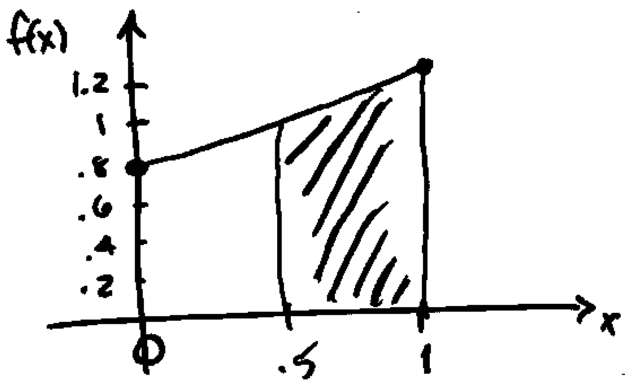
(11)

$$P(1 < X < 2) = F(2) - F(1)$$

$$= 1 - e^{-2/4} - \left(1 - e^{-\frac{1}{4}}\right)$$

$$= e^{-1/4} - e^{-1/2}$$

Example: $f(x) = \begin{cases} \frac{2}{5}(x+2) & 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$



(12)

$$P(X > .5) = P(.5 < X < 1)$$

$$= \int_{.5}^{1} f(x)\,dx = \int_{.5}^{1} \frac{2}{5}(x+2)\,dx$$

$$= \frac{2}{5}\left(\frac{x^2}{2} + 2x\right)\Big|_{.5}^{1}$$

$$= \frac{2}{5}\left[\frac{1}{2} + 2 - \left(\frac{1}{8} + 1\right)\right]$$

$$= \frac{2}{5}\left[\frac{11}{8}\right] = \frac{11}{20}$$

Hw #3 due 4/24

p.65 # 80, 86, 98

p.72 # 102, 104

p.88 # 8, 18, 24