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Random variables

Defn: A random variable is a variable whose value is determined by the outcome of an experiment.

Actually, a random variable is a function whose domain is S and whose range is a subset of $\mathbb{R}$.

Example: Flip 2 coins. Let $X = \#\text{heads}$

$$S = \left\{ \begin{array}{l} HH \to 2 \\ HT \to 1 \\ TH \to 1 \\ TT \to 0 \end{array} \right.$$

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If the range of X is finite or countably infinite, then X is called a discrete random variable.

If the range of X is an interval then X is called a continuous random variable.

Back to the example:

x	$p(x)$
0	$1/4$
1	$1/2$
2	$1/4$
	1

$$p(0) = P(X=0) = P(\{TT\}) = \frac{1}{4}$$

$$p(1) = P(X=1) = P(\{HT, TH\}) = 2/4$$

The probability distribution of a discrete random variable X consists of $(x, p(x))$ for all possible values.

Example: Roll 2 dice. Let X = sum

x	$p(x)$
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36
	1

Let Y = minimum

y	$p(y)$
1	11/36
2	9/36
3	7/36
4	5/36
5	3/36
6	1/36
	1

Note $0 \leq p(x) \leq 1$

and $\sum_x p(x) = 1$

Example: Draw 3, without replacement, from a deck of 52

Let X = number of red cards

x	$p(x)$
0	.1176
1	.3824
2	.3824
3	.1176
	1

If we keep track of order

$$n(S) = P_3^{52}$$

$$= \frac{52!}{49!}$$

$$= 52 \cdot 51 \cdot 50$$

$$p(0) = P(X=0) = \frac{n(\text{event})}{n(S)}$$

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$n(X=0) = 26\cdot 25\cdot 24$

$$p(0) = \frac{26\cdot 25\cdot 24}{52\cdot 51\cdot 50}$$

Example 6: Observe 4 independent components in a system. Each one has a 20% chance of failing.

X = # of components that fail.

x	$p(x)$
0	$(.8)^4$
1	$4(.8)^3(.2)$
2	$6(.8)^2(.2)^2$
3	$4(.8)(.2)^3$
4	$(.2)^4$

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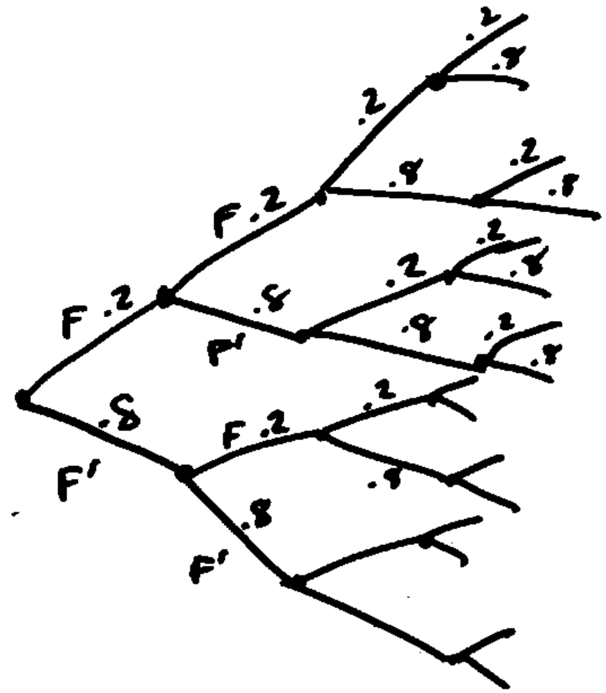

$p(0) = P(X=0) = (.8)^4$

$p(4) = P(X=4) = (.2)^4$

$p(1) = P(X=1) = 4\,(.8)^3(.2)^1$

$p(3) = P(X=3) = 4(.8)^1(.2)^3$

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$p(2) = P(X=2) = 6(.8)^2(.2)^2$

$$\left[\begin{array}{l} p(x) = \binom{4}{x}(.2)^x(.8)^{4-x} \\ x = 0,1,2,3,4 \end{array}\right]$$ Prob. Distr.

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} \end{array}$$

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Example Fire 10 independent shots at a target. Each shot has a 70% chance of hitting the target. X = # hits.

$$\left[\begin{array}{l} p(x) = \binom{10}{x}(.7)^x(.3)^{10-x} \\ X = 0,1,\ldots,10 \end{array}\right]$$

Some characteristics of distributions:

$$\mu = E[X] = \sum_x x\,p(x)$$

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Definition of expectation:

Let X be a random variable and $g(X)$ be any function of X.

Then $E[g(X)] = \sum_x g(x)\,p(x)$

For example, $E[\sqrt{X}] = \sum_x \sqrt{x}\,p(x)$

or $E[X^2] = \sum_x x^2\,p(x)$

Defn: The variance of a discrete r.v. X is
$\sigma^2 = E[(X-\mu)^2] = \sum_x (x-\mu)^2 p(x)$

$= V(X)$ or $Var(X)$

(10)

2 coins example : Find μ, σ^2

x	$p(x)$
0	1/4
1	1/2
2	1/4

$\mu = \sum x\,p(x)$
$= 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4}$
$= 0 + \frac{1}{2} + \frac{1}{2}$
$= 1$

$\sigma^2 = E[(X-\mu)^2]$
$= \sum (x-\mu)^2 p(x)$
$= (0-1)^2\cdot\frac{1}{4} + (1-1)^2\cdot\frac{1}{2} + (2-1)^2\cdot\frac{1}{4} = \frac{1}{2}$

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deviation of the random variable

$$\sigma = \sqrt{\tfrac{1}{2}} = .707$$

Another Baye's example

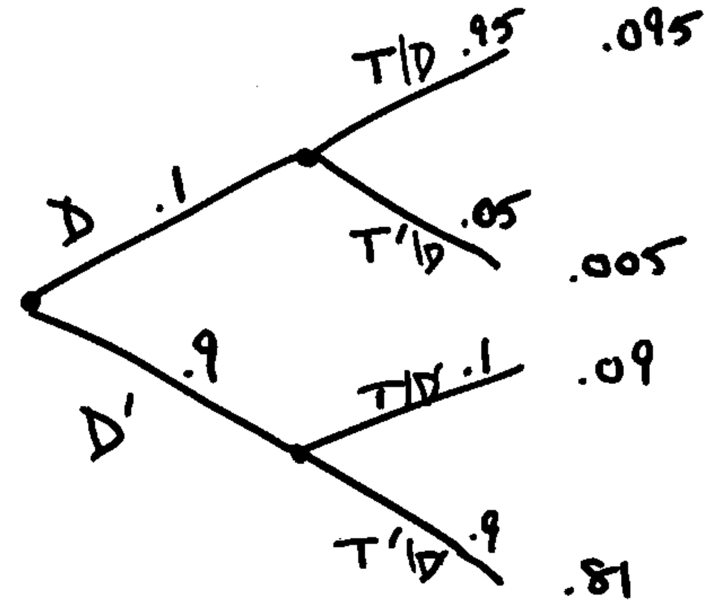
D: using cocaine
T: testing positive

Suppose that 10% are users.

If a person is using the drug, the test will show a positive result 95% (false negative 5%)

If a person is not using it, the test will show a negative 90% (false pos. 10%)

(12)



	T	T'	
D	.095	.005	.1
D'	.09	.81	.9
	.185	.815	1

$$P(D \mid T) = \frac{P(D \cap T)}{P(T)} = \frac{.095}{.185} = .514$$