451

1

4-10

Some additional counting rules

n objects of k types

n_1 are of type 1
n_2 " " " 2
$\vdots$
n_k " " " k

$n_1 + n_2 + \dots + n_k = n$

How many different ways can these items be arranged (or ranked, or ordered)?

__Example:__ AAA BB C

If we could keep track of the ordering of the individual As and Bs, then $6!$

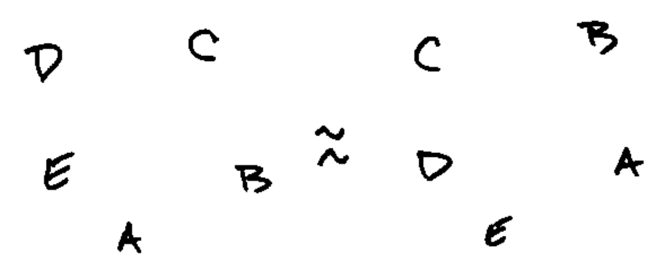
2

$$\frac{6!}{3!\,2!\,1!}$$

The general answer is $\dfrac{n!}{n_1!\,n_2!\cdots n_k!}$

$$= \binom{n}{n_1, n_2, \dots, n_k}$$

How many different ways can n items be arranged in a circle?



$$\frac{5!}{5} = 4!$$

(3)

In general, the answer is $(n-1)!$

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neither A nor B

$$A' \cap B' = (A \cup B)'$$

(De Morgan's Law)

Conditional probability

		Salary			
		High	Mid	Low	
Gender	M	30	40	30	100
	F	60	50	40	150
		90	90	70	250

(4)

Experiment: Select 1 person from the 250.

$$n(S) = 250$$

Let M be the event that the person selected is male.

$$n(M) = 100$$

$$P(M) = \frac{100}{250} = .4$$

Let H be the event that the person selected has a high salary.

$$n(H) = 90$$

$$P(H) = \frac{90}{250} = .36$$

5

Find $P(M \cap H) = \dfrac{n(M \cap H)}{n(S)}$

$$= \frac{30}{250} = .12$$

Given that the person selected is male, what is the probability that he has a high salary?

This is equivalent to restricting the sample space to males only.

$$P(H \mid M) = \frac{30}{100} = \frac{n(H \cap M)}{n(M)}$$

$\uparrow$ given

6

The conditional probability of A, given B, is

$$P(A \mid B) = \frac{n(A \cap B)}{n(B)}$$

Note: $\dfrac{n(A \cap B) \div n(S)}{n(B) \div n(S)} = \dfrac{P(A \cap B)}{P(B)}$

Using the 2^{nd} version of the definition,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$\Longrightarrow \quad P(A \cap B) = P(B)P(A \mid B)$$

7

Experiment : Draw 2 cards from a deck, without replacement

Find the probability that they are both aces

Hard solution 1: keep track of order

$$n(S) = P_2^{52}$$

$$= \frac{52!}{50!} = 52 \cdot 51$$

A is the event that both cards are aces

$$n(A) = P_2^4 = \frac{4!}{2!} = 4 \cdot 3$$

$$P(A) = \frac{4.3}{52 \cdot 51} = .0045$$

8

Hard solution 2: ignore order

$$n(S) = \binom{52}{2} = \frac{52!}{2!\,50!}$$

$$= \frac{52 \cdot 51}{2} = 26 \cdot 51$$

$$n(A) = \binom{4}{2} = \frac{4!}{2!\,2!} = \frac{4.3}{2} = 6$$

$$P(A) = \frac{6}{26 \cdot 51} = .0045$$

Easy solution : Let A_1 be the event that the 1st card was an ace, + A_2 be the event that the 2nd card was an ace

"both" : $A_1 \cap A_2$

9

$$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2 | A_1)$$
$$= \frac{4}{52} \cdot \frac{3}{51}$$

In the gender/salary problem,

$$P(H|M) = \frac{30}{100} = .3$$

Also $P(H) = \frac{90}{250} = .36$

Notice that $P(H|M) \neq P(H)$

<u>Defn</u>: Events A and B are independent if $P(A|B) = P(A)$

10

Now notice that if A and B are independent,

$$P(A \cap B) = P(A)\,P(B)$$

<u>Example</u>: You are blindfolded & put on a train to either Seattle or San Francisco. Your friends tell you that there is a 40% chance of Seattle & 60% chance of S.F.

If the destination is Seattle, there is a 90% chance of seeing Starbucks on the first street corner.

11

If the destination is SF, there is a 50% chance of Starbucks.

If you see a Starbucks, what is the probability that you are in Seattle?

Let A: Seattle $\quad P(A) = .4$

so A': SF $\quad P(A') = .6$

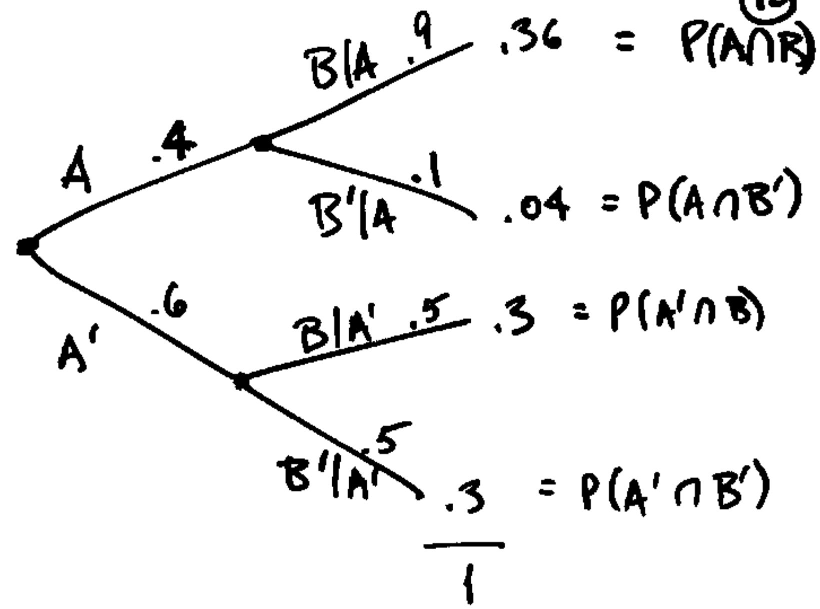
Let B: Starbucks

$P(B|A) = .9$

$P(B|A') = .5$

Question: $P(A|B)$

12



	B	B'	
A	.36	.04	.40
A'	.30	.30	.60
	.66	.34	1

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.36}{.66} = .55$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{P(A\cap B) + P(A'\cap B)}$$

$$= \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A')P(B|A')}$$

"Bayes' Rule"

HW #2 due 4-17

p.39 # 14, 16

p.48 # 30, 32, 38, 48, 50

p.56 # 58, 60, 64