Chapter 2

Probability

451
1
4-5

Experiment: A process that leads to one of several possible outcomes

Sample space (S): set of all possible outcomes of an experiment

Example 1: The experiment is to flip a coin

$S = \{H, T\}$

2

Example 2: Roll a die

$S = \{1, 2, 3, 4, 5, 6\}$

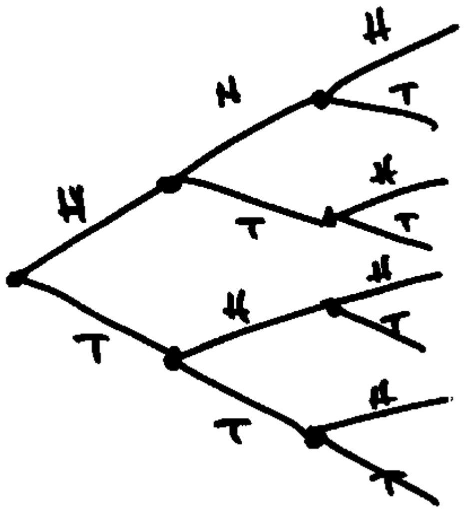
Example 3: Flip 2 coins in sequence

$S = \{HH, HT, TH, TT\}$

Example 4: Flip 3 coins in sequence

$S = \{HHH, HTH, HHT, HTT, THH, THT, TTH, TTT\}$

Tree diagram



Event: A subset of the sample space

Example 3: (2 coins)

Let A be the event that both coins are heads. $A = \{HH\}$

Let B be the event that exactly 1 coin is heads.

$$B = \{HT, TH\}$$

Probability (for finite sample spaces)

$$P(A) = \frac{n(A)}{n(S)}$$

In the previous example,

$$P(\text{both heads}) = P(A) = \frac{1}{4}$$

$$P(\text{exactly 1 head}) = P(B) = \frac{2}{4} = \frac{1}{2}$$

Example 5: Roll 2 dice, in sequence.

Find the probability of getting a sum of 9.

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & & & & \vdots \\ (3,1) & & & & & \vdots \\ (4,1) & & & & & \vdots \\ (5,1) & & & & & \vdots \\ (6,1) & \cdots & & & & (6,6) \end{array} \right\}$$

$n(S) = 36$

$A = \{(3,6), (6,3), (5,4), (4,5)\}$

$n(A) = 4 \qquad P(A) = 4/36 = \frac{1}{9}$

Counting Rules

1 Multiplication principle

If a process can be broken down into a sequence of operations, then the total number of possible outcomes is the product of the number of outcomes at each stage

2 Factorial Rule

The number of ways of ordering n items is $n! = n(n-1)(n-2)\cdots 1$

(7)

5 people

$$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$$

$$= 5!$$

(3) Permutation rule

Start with n items. Select and rank r of them. The number of possible outcomes is

$$P_r^n = n(n-1) \cdots (n-r+1) = {}_nP_r$$

5 finalists. Award 1st, 2nd, 3rd place

$$\underline{5} \times \underline{4} \times \underline{3} = 60 = P_3^5$$

(8)

Notice:

$$P_r^n = \frac{n(n-1) \cdots (n-r+1)(n-r) \cdots 1}{(n-r) \cdots 1}$$

$$= \frac{n!}{(n-r)!}$$

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20 people in a club

Elect Pres, VP, Sec, Treas.

Number of outcomes is P_4^{20}

$$P_4^{20} = \frac{20!}{16!} = 20 \cdot 19 \cdot 18 \cdot 17$$

9

(4) Combination rule

The number of ways of selecting r items from a group of n items, without regard to order, is

$$ {}_nC_r = C^n_r = \binom{n}{r} = \frac{P^n_r}{r!} $$

$$ = \frac{n!}{r!(n-r)!} $$

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5 items ABCDE

Select 3, order not important

ABC	ACD	BCD	CDE
ABD	ACE	BCE	
ABE	ADE	BDE	

$$ 10 = \frac{P^5_3}{3!} $$

10

20 members of a club

Need a committee of 4

How many different outcomes?

$$ \binom{20}{4} $$

Deal 5 cards from a deck of 52, order unimportant

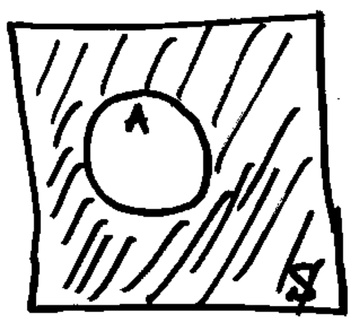
$$ \# \text{ outcomes} = \binom{52}{5} $$

$$ n(S) = 2,598,960 $$

Let A be the event of getting a royal flush $n(A) = 4$

$$P(A) = \frac{4}{2{,}598{,}960}$$

The <u>complement</u> of an event A is the set A′ of items in S that are <u>not</u> in the event.



A′

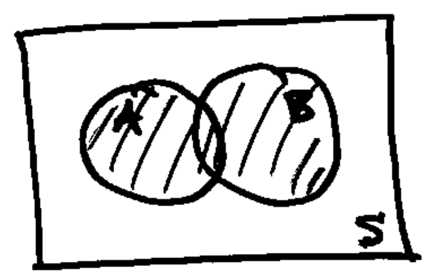
$$n(A) + n(A') = n(S)$$

$$\frac{n(A)}{n(S)} + \frac{n(A')}{n(S)} = \frac{n(S)}{n(S)}$$

$$P(A) + P(A') = 1$$

<u>Complement Rule</u>

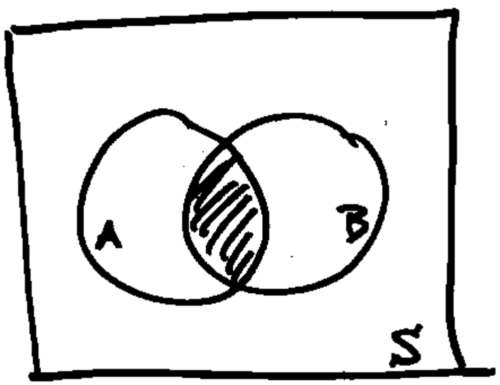
The <u>union</u> of two events A, B is $A \cup B$, which is the set of items contained in either A <u>or</u> in B (or both).



$A \cup B$

(13)

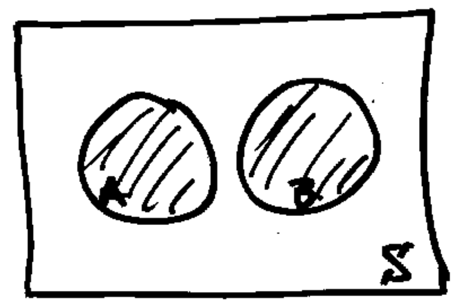
The intersection of two events A, B is $A \cap B$, which is the set of items which appear in both A <u>and</u> B.



$A \cap B$

(14)

Two events A and B are <u>disjoint</u> or <u>mutually exclusive</u> if their intersection is empty, i.e. $A \cap B = \emptyset = \{\}$

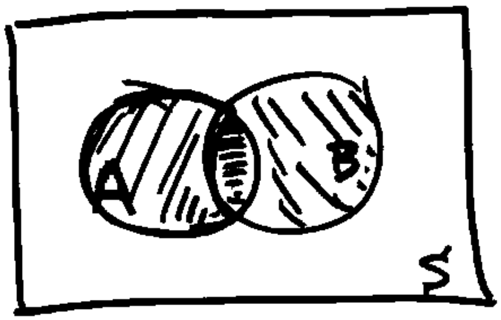


$A \cup B$ for disjoint sets

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$$

Union rule:
$P(A \cup B) = P(A) + P(B)$ for disjoint sets

In the general case,



$$A \cup B = [A \cap B'] \cup [A \cap B] \cup [A' \cap B]$$

These 3 pieces are disjoint

$$P(A \cup B) = \underline{P(A \cap B') + P(A \cap B)} + \underline{P(A' \cap B)}$$

using the union rule for disjoint sets

Similarly,

$$P(A) = \underline{P(A \cap B') + P(A \cap B)}$$

Also,

$$P(B) = P(A \cap B) + P(A' \cap B)$$

From the last equation,

$$P(A' \cap B) = \underline{\underline{P(B) - P(A \cap B)}}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Union rule