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Name: KEY

Midterm Exam

Stat 4/551 | **Fall 2016** | **November 3**

(24) 1. Consider the following 21 job satisfaction scores given by a random sample of factory workers:

56	24	75	52
59	67	80	38
81	66	62	51
77	82	39	67
60	69	81	33
60			

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2 | 4
3 | 983
4 |
5 | 6921
6 | 0076927
7 | 75
8 | 1201
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(3) a) $\bar{x}$ = 60.9.

(3) b) median = 62.

(3) c) mode = 60, 67, 81.

(3) d) Range = 58. 82-24

(3) e) s = 16.7.

(3) f) Q1 = 52.

(3) g) Q3 = 75.

(3) h) Draw the box plot and comment on the shape. → Skewed to the left!

IQR = 75 - 52 = 23

low fence = 52 - 1.5(23) = 17.5

high fence = 75 + 1.5(23) = 109.5 No outliers

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2 | 4
3 | 389
4 |
5 | 1269
6 | 00267 79
7 | 57
8 | 0112
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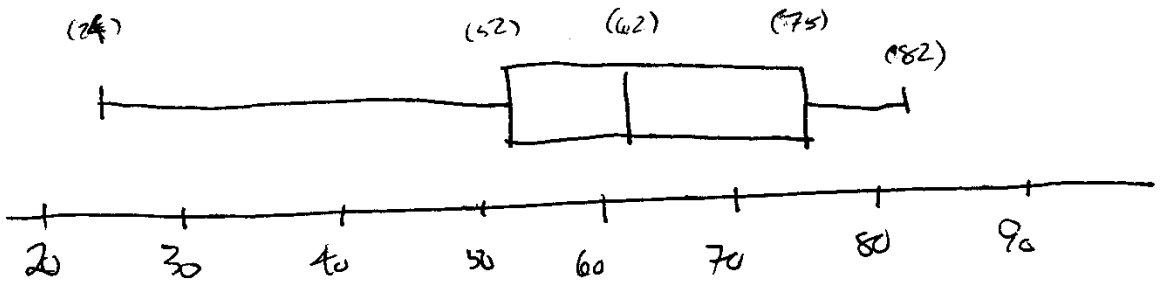

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2a) On the average, your web site is viewed once every 20 seconds. Find the probability that, in a 1-minute interval, there are no visitors to the site. (5)

Poisson($\mu=3$) $p(x) = \frac{3^x e^{-3}}{x!}$

$P(0) = \frac{3^0 e^{-3}}{0!} = \boxed{.0498}$

b) Suppose that a company wins 20% of the contracts on which it bids. Assume independence between bids. In one month, it bids on 10 contracts. Find the probability that it wins exactly 5 of them. (5)

Bino($n=10$, $p=.2$) $p(x) = \binom{10}{x}(.2)^x(.8)^{10-x}$

$p(5) = \binom{10}{5}(.2)^5(.8)^5 = \boxed{.0264}$

c) Suppose that a production batch of 64 items contains 8 defective units. You take a random sample of 12 items. Find the probability that your sample contains at least one defective unit. (5)

Hypergeom($N=64$, $N_1=8$, $n=12$) $p(x) = \frac{\binom{8}{x}\binom{56}{12-x}}{\binom{64}{12}}$

$P(X \geq 1) = 1 - p(0) = 1 - \frac{\binom{8}{0}\binom{56}{12}}{\binom{64}{12}} = \boxed{.8300}$

d) Assume that speeds on I405 are normally distributed with mean 52mph and standard deviation 5mph. What percentage of the vehicles are travelling over 60mph? (5)

$N(\mu=52, \sigma=5)$

$P(X>60) = P(z > \frac{60-52}{5}) = P(z > 1.6) = \boxed{.0548}$

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3. Each day, 10% of the email sent to my address is spam. If an email is actually spam, the spam filter will correctly identify it 90% of the time. If the email is not spam, the spam filter still tags it as spam 5% of the time.

(10) a) Complete the table with the correct joint and marginal probabilities:

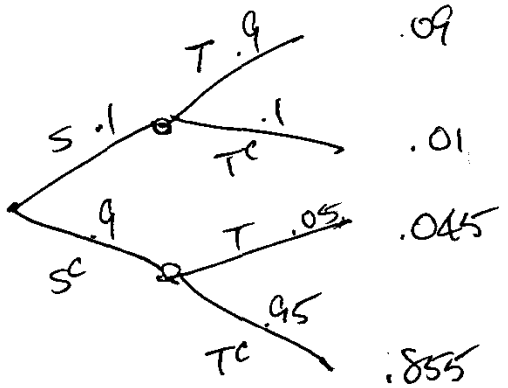
		ID'ed as:		
		Spam	Not spam	Total
Actually:	Spam	.09	.01	.1
	Not spam	.045	.855	.9
	Total	.135	.865	1

(10) b) Given that a message has been tagged as spam, what is the probability that it is actually spam?

$$P(S|T) = \frac{P(S \cap T)}{P(T)}$$

$$= \frac{.09}{.135}$$

$$= \boxed{.667}$$



S = "spam"

T = "tagged as spam"

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4a) In a marketing study, the participants must select their favorite 5 movies out of a list of 15 movies that appear on a list. How many different outcomes are possible?

6

$\binom{15}{5} = \boxed{3003}$

b) How many ways are there of arranging the letters in the word ENGINEERING?

6

$\frac{11!}{3!\,3!\,2!\,2!\,1!} = \boxed{277,200}$

3 E's
3 N's
2 G's
2 I's
1 R

c) There are 10 finalists competing in a contest that awards distinct prizes for first, second, and third place. How many different outcomes are possible?

6

$P(10,3) = \frac{10!}{7!} = \boxed{720}$

18 points Name: KEY

Detach this page and take home. Complete and turn in at next class.

5. The following is the joint density function of the random variables X and Y:

$$f(x,y)=\begin{cases} c(x^2+y^2), & 0<x<1, \quad 0<y<1 \\ 0 & \text{elsewhere} \end{cases}.$$

3 a) Find c.
3 b) Find $g(x)$ and $h(y)$.
3 c) Find $E(X)$ and $E(Y)$.
3 d) Find $\text{Var}(X)$ and $\text{Var}(Y)$.
3 e) Find $E(XY)$ and $\text{Cov}(X,Y)$.
3 f) Find ρ_{XY}.

a) $c\int_0^1\int_0^1 (x^2+y^2)\,dy\,dx = c\int_0^1 \left(x^2y+\frac{y^3}{3}\right)\Big|_{y=0}^{1} dx = c\int_0^1 \left(x^2+\frac{1}{3}\right)dx$

$= c\left(\frac{x^3}{3}+\frac{1}{3}x\right)\Big|_0^1 = c\cdot\frac{2}{3} \overset{\text{set}}{=} 1 \quad \therefore \boxed{c=3/2}$

b) $g(x)=\int_0^1 \frac{3}{2}(x^2+y^2)\,dy = \frac{3}{2}\left(x^2y+\frac{y^3}{3}\right)\Big|_{y=0}^{1} = \frac{3}{2}\left(x^2+\frac{1}{3}\right), \quad 0<x<1$

Similarly, $\boxed{h(y)=\frac{3}{2}\left(y^2+\frac{1}{3}\right), \quad 0<y<1}$

c) $E[X]=\int_0^1 x\cdot\frac{3}{2}\left(x^2+\frac{1}{3}\right)dx = \frac{3}{2}\int_0^1 x^3+\frac{x}{3}\,dx = \frac{3}{2}\left(\frac{x^4}{4}+\frac{x^2}{6}\right)\Big|_0^1$

$= \frac{3}{2}\left(\frac{1}{4}+\frac{1}{6}\right) = \frac{3}{2}\cdot\frac{5}{12} = \frac{5}{8}$ Also $\boxed{E[Y]=5/8}$

d) $E[X^2]=\int_0^1 x^2\cdot\frac{3}{2}\left(x^2+\frac{1}{3}\right)dx = \frac{3}{2}\int_0^1 x^4+\frac{x^2}{3}\,dx = \frac{3}{2}\left(\frac{x^5}{5}+\frac{x^3}{9}\right)\Big|_0^1$

$= \frac{3}{2}\left(\frac{1}{5}+\frac{1}{9}\right) = \frac{3}{2}\cdot\frac{14}{45} = \frac{7}{15}$ Also $E[Y^2]=7/15$

$\sigma_X^2 = \frac{7}{15}-\left(\frac{5}{8}\right)^2 = \frac{448-375}{960} = \frac{73}{960}$ $\boxed{\text{Also } \sigma_Y^2=\frac{73}{960}}$ 0.076

e) $E[XY]=\int_0^1\int_0^1 xy\cdot\frac{3}{2}(x^2+y^2)\,dy\,dx = \frac{3}{2}\int_0^1\int_0^1 \left(x^3y+xy^3\right)dy\,dx$

$= \frac{3}{2}\int_0^1 \left(\frac{x^3y^2}{2}+\frac{xy^4}{4}\right)\Big|_{y=0}^{1} dx = \frac{3}{2}\int_0^1 \frac{x^3}{2}+\frac{x}{4}\,dx = \frac{3}{2}\left(\frac{x^4}{8}+\frac{x^2}{8}\right)\Big|_0^1 = \frac{3}{2}\cdot\frac{2}{8} = \boxed{\frac{3}{8}}$

(over)