# Stat 451 Homework #8

6.24 $\mu = np = (400)(1/2) = 200$, $\sigma = \sqrt{npq} = \sqrt{(400)(1/2)(1/2)} = 10$.

(a) $z_1 = (184.5 - 200)/10 = -1.55$ and $z_2 = (210.5 - 200)/10 = 1.05$.
$P(184.5 < X < 210.5) = P(-1.55 < Z < 1.05) = 0.8531 - 0.0606 = 0.7925$.

(b) $z_1 = (204.5 - 200)/10 = 0.45$ and $z_2 = (205.5 - 200)/10 = 0.55$.
$P(204.5 < X < 205.5) = P(0.45 < Z < 0.55) = 0.7088 - 0.6736 = 0.0352$.

(c) $z_1 = (175.5 - 200)/10 = -2.45$ and $z_2 = (227.5 - 200)/10 = 2.75$.
$P(X < 175.5) + P(X > 227.5) = P(Z < -2.45) + P(Z > 2.75)$
$= P(Z < -2.45) + 1 - P(Z < 2.75) = 0.0071 + 1 - 0.9970 = 0.0101$.

6.42 (a) $P(X < 1) = 4\int_0^1 xe^{-2x}\,dx = [-2xe^{-2x} - e^{-2x}]\big|_0^1 = 1 - 3e^{-2} = 0.5940$.

(b) $P(X > 2) = 4\int_0^\infty xe^{-2x}\,dx = [-2xe^{-2x} - e^{-2x}]\big|_2^\infty = 5e^{-4} = 0.0916$.

6.46 $P(X < 1) = \frac{1}{2}\int_0^1 e^{-x/2}\,dx = -e^{-x/2}\big|_0^1 = 1 - e^{-1/2} = 0.3935$. Let $Y$ be the number of switches that fail during the first year. Using the normal approximation we find $\mu = (100)(0.3935) = 39.35$, $\sigma = \sqrt{(100)(0.3935)(0.6065)} = 4.885$, and $z = (30.5 - 39.35)/4.885 = -1.81$. Therefore, $P(Y \le 30) = P(Z < -1.81) = 0.0352$.

6.52 $\alpha\beta = 10$; $\sigma = \sqrt{\alpha\beta^2}\sqrt{50} = 7.07$.

(a) Using integration by parts,

$$P(X \le 50) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^{50} x^{\alpha-1} e^{-x/\beta}\,dx = \frac{1}{25}\int_0^{50} xe^{-x/5}\,dx = 0.9995.$$

(b) $P(X < 10) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^{10} x^{\alpha-1} e^{-x/\beta}\,dx$. Using the incomplete gamma with $y = x/\beta$, we have

$$P(X < 10) = P(Y < 2) = \int_0^2 ye^{-y}\,dy = 0.5940.$$