Stat 451 Homework #5

4.58 $E(X) = (2)(0.40) + (4)(0.60) = 3.20$, and
$E(Y) = (1)(0.25) + (3)(0.50) + (5)(0.25) = 3$. So,

(a) $E(2X - 3Y) = 2E(X) - 3E(Y) = (2)(3.20) - (3)(3.00) = -2.60.$

(b) $E(XY) = E(X)E(Y) = (3.20)(3.00) = 9.60.$

4.62 $\mu = 52$ and $\sigma = 6.5$. Solving $\mu + k\sigma = 71.5$ we obtain $k = 3$. So,

$$P(\mu - 3\sigma < X < \mu + 3\sigma) \geq 1 - \frac{1}{3^2} = 0.8889,$$

which is

$$P(32.5 < X < 71.5) \geq 0.8889.$$

we obtain $P(X > 71.5) < \frac{1-0.8889}{2} = 0.0556$ using the symmetry.

4.70 $E(Z) = E(XY) = E(X)E(Y) = \int_0^1 \int_2^\infty 16xy(y/x^3)\, dx\, dy = 8/3.$

4.72 $\mu_X = \mu_Y = 3.5$. $\sigma_X^2 = \sigma_Y^2 = [(1)^2 + (2)^2 + \cdots + (6)^2](1/6) - (3.5)^2 = \frac{35}{12}$.

(a) $\sigma_{2X-Y} = 4\sigma_X^2 + \sigma_Y^2 = \frac{175}{12}$;

(b) $\sigma_{X+3Y-5} = \sigma_X^2 + 9\sigma_Y^2 = \frac{175}{6}$.