Stat 451 Homework #3

2.80 Consider the events:
A: a person is experiencing hypertension,
B: a person is a heavy smoker,
C: a person is a nonsmoker.

(a) $P(A \mid B) = 30/49$;

(b) $P(C \mid A') = 48/93 = 16/31$.

2.86 Consider the events:
H: the husband will vote on the bond referendum,
W: the wife will vote on the bond referendum.
Then $P(H) = 0.21$, $P(W) = 0.28$, and $P(H \cap W) = 0.15$.

(a) $P(H \cup W) = P(H) + P(W) - P(H \cap W) = 0.21 + 0.28 - 0.15 = 0.34$.

(b) $P(W \mid H) = \frac{P(H \cap W)}{P(H)} = \frac{0.15}{0.21} = \frac{5}{7}$.

(c) $P(H \mid W') = \frac{P(H \cap W')}{P(W')} = \frac{0.06}{0.72} = \frac{1}{12}$.

2.98 $P = (0.95)[1 - (1 - 0.7)(1 - 0.8)](0.9) = 0.8037$.

2.102 Let S_1, S_2, S_3, and S_4 represent the events that a person is speeding as he passes through the respective locations and let R represent the event that the radar traps is operating resulting in a speeding ticket. Then the probability that he receives a speeding ticket:
$P(R) = \sum_{i=1}^{4} P(R \mid S_i)P(S_i) = (0.4)(0.2) + (0.3)(0.1) + (0.2)(0.5) + (0.3)(0.2) = 0.27$.

2.104 $P(S_2 \mid R) = \frac{P(R \cap S_2)}{P(R)} = \frac{0.03}{0.27} = 1/9$.

3.8 Referring to the sample space in Exercise 3.3 and making use of the fact that $P(H) = 2/3$ and $P(T) = 1/3$, we have
$P(W = -3) = P(TTT) = (1/3)^3 = 1/27$;
$P(W = -1) = P(HTT) + P(THT) + P(TTH) = 3(2/3)(1/3)^2 = 2/9$;
$P(W = 1) = P(HHT) + P(HTH) + P(THH) = 3(2/3)^2(1/3) = 2/9$;
$P(W = 3) = P(HHH) = (2/3)^3 = 8/27$;
The probability distribution for W is then

w	-3	-1	1	3
$P(W = w)$	$1/27$	$2/9$	$2/9$	$8/27$

3.18 (a) $P(X < 4) = \int_2^4 \frac{2(1+x)}{27}\, dx = \left.\frac{(1+x)^2}{27}\right|_2^4 = 16/27.$

(b) $P(3 \le X < 4) = \int_3^4 \frac{2(1+x)}{27}\, dx = \left.\frac{(1+x)^2}{27}\right|_3^4 = 1/3.$

3.24 There are $\binom{10}{4}$ ways of selecting any 4 CDs from 10. We can select x jazz CDs from 5 and $4 - x$ from the remaining CDs in $\binom{5}{x}\binom{5}{4-x}$ ways. Hence

$$f(x) = \frac{\binom{5}{x}\binom{5}{4-x}}{\binom{10}{4}}, \qquad x = 0, 1, 2, 3, 4.$$