Stat 451 Homework #2

2.14 (a) $A \cup C = \{0, 2, 3, 4, 5, 6, 8\}$.

(b) $A \cap B = \phi$.

(c) $C' = \{0, 1, 6, 7, 8, 9\}$.

(d) $C' \cap D = \{1, 6, 7\}$, so $(C' \cap D) \cup B = \{1, 3, 5, 6, 7, 9\}$.

(e) $(S \cap C)' = C' = \{0, 1, 6, 7, 8, 9\}$.

(f) $A \cap C = \{2, 4\}$, so $A \cap C \cap D' = \{2, 4\}$.

2.16 (a) $M \cup N = \{x \mid 0 < x < 9\}$.

(b) $M \cap N = \{x \mid 1 < x < 5\}$.

(c) $M' \cap N' = \{x \mid 9 < x < 12\}$.

2.30 With $n_1 = 2$ choices for the first question, $n_2 = 2$ choices for the second question, and so forth, the generalized multiplication rule yields $n_1 n_2 \cdots n_9 = 2^9 = 512$ ways to answer the test.

2.32 (a) By Theorem 2.3, $7! = 5040$.

(b) Since the first letter must be m, the remaining 6 letters can be arranged in $6! = 720$ ways.

2.38 (a) $8! = 40320$.

(b) There are 4! ways to seat 4 couples and then each member of a couple can be interchanged resulting in $2^4(4!) = 384$ ways.

(c) By Theorem 2.3, the members of each gender can be seated in 4! ways. Then using Theorem 2.1, both men and women can be seated in $(4!)(4!) = 576$ ways.

2.48 $\binom{9}{1,4,4} + \binom{9}{2,4,3} + \binom{9}{1,3,5} + \binom{9}{2,3,4} + \binom{9}{2,2,5} = 4410.$

2.50 Assume February 29th as March 1st for the leap year. There are total 365 days in a year. The number of ways that all these 60 students will have different birth dates (i.e, arranging 60 from 365) is ${}_{365}P_{60}$. This is a very large number.

2.58 (a) Let A = Defect in brake system; B = Defect in fuel system; $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.17 - 0.15 = 0.27$.

(b) $P(\text{No defect}) = 1 - P(A \cup B) = 1 - 0.27 = 0.73$.

2.60 (a) Of the $(6)(6) = 36$ elements in the sample space, only 5 elements (2,6), (3,5), (4,4), (5,3), and (6,2) add to 8. Hence the probability of obtaining a total of 8 is then 5/36.

(b) Ten of the 36 elements total at most 5. Hence the probability of obtaining a total of at most is 10/36=5/18.

2.64 Any four of a kind, say four 2's and one 5 occur in $\binom{5}{1} = 5$ ways each with probability $(1/6)(1/6)(1/6)(1/6)(1/6) = (1/6)^5$. Since there are ${}_6P_2 = 30$ ways to choose various pairs of numbers to constitute four of one kind and one of the other (we use permutation instead of combination is because that four 2's and one 5, and four 5's and one 2 are two different ways), the probability is $(5)(30)(1/6)^5 = 25/1296$.