Stat 451
2-8-18
①

Joint probability distributions

Discrete case

Example: Roll 2 dice. Let X = minimum value
Y = maximum value

		Y: 1	2	3	4	5	6	
X	1	1/36	2/36	2/36	2/36	2/36	2/36	11/36
	2	0	1/36	2/36	2/36	2/36	2/36	9/36
	3	0	0	1/36	2/36	2/36	2/36	7/36
	4	0	0	0	1/36	2/36	2/36	5/36
	5	0	0	0	0	1/36	2/36	3/36
	6	0	0	0	0	0	1/36	1/36
		1/36	3/36	5/36	7/36	9/36	11/36	1

②

The body of the table contains the joint probability distribution $p(x,y) = P(X=x \cap Y=y)$

$$p(3,4) = P(X=3 \cap Y=4) = 2/36$$

$$P(X=x) = \sum_y p(x,y)$$

and $P(Y=y) = \sum_x p(x,y)$

Defn: X & Y are independent if

$$p(x,y) = p_X(x)\, p_Y(y) \quad \forall x,y$$

$p(5,6) = 2/36$

$p_X(5) = 3/36 \qquad p_Y(6) = 11/36$

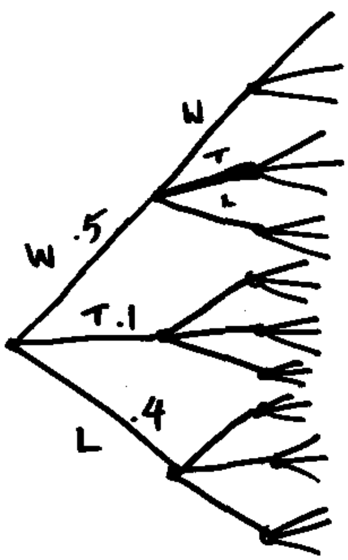
Check: $2/36 \stackrel{?}{=} \frac{3}{36} \cdot \frac{11}{36}$ No

$\therefore$ X & Y are not independent

Example A team plays 3 games with independent outcomes

$P(\text{win}) = .5 \qquad P(\text{lose}) = .4 \qquad P(\text{tie}) = .1$

$X = \#\text{wins} \qquad Y = \#\text{losses}$



Y (losses)

X (wins)	0	1	2	3	
0	$(.1)^3$	$3(.1)^2(.4)$	$3(.1)(.4)^2$	$(.4)^3$	.125
1	$3(.5)(.1)^2$	$6(.5)(.1)(.4)$	$3(.5)(.4)^2$	0	.375
2	$3(.5)^2(.1)$	$3(.5)^2(.4)$	0	0	.375
3	$(.5)^3$	0	0	0	.125
	.216	.432	.288	.064	1

Find the conditional probability of getting 3 wins, given that there are no losses.

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$$P(X=3 \mid Y=0) = \frac{P(X=3 \cap Y=0)}{P(Y=0)}$$

$$= \frac{p(3,0)}{p_Y(0)} = \frac{(.5)^3}{.216}$$

$$= .579$$

The pattern was

$$p(x,y) = \frac{3!}{x!\,y!\,(3-x-y)!}(.5)^x(.4)^y(.1)^{3-x-y},$$

$$0 \le x+y \le 3$$

(this is a trinomial distribution)

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Midterm Exam (Tues 2/13)

1-page-of-notes (front & back)

Calculator

- Descriptive statistics
- Probability: marginal, joint, conditional, independence
- Counting rules
- Bernoulli, Binomial, Hypergeometric, Geometric, Neg Bino, Poisson
- Uniform, Exponential, Normal

From HW#2

#6 $P(E_1) = .95$

$P(E_2) = .92$

$P(E_1 \cap E_2) = .88$

a) $P(E_1 \cap E_2^c) = P(E_1 - E_2)$

$= P(E_1) - P(E_1 \cap E_2) = .95 - .88 = .07$

b) $P(E_2 \cap E_1^c) = P(E_2 - E_1)$

$= P(E_2) - P(E_2 \cap E_1) = .92 - .88 = .04$

c) $P\left((E_1 \cap E_2^c) \cup (E_2 \cap E_1^c)\right)$

$= P(E_1 \cap E_2^c) + P(E_2 \cap E_1^c) = .07 + .04 = .11$

d) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= .95 + .92 - .88 = .99$