Stat 451
1-18-18
(1)

Conditional probability

"Contingency table"
"Cross-tab"

Gender \ Income	H High	Mid	Low	
M	30	40	30	100
F	60	50	40	150
	90	90	70	250

Experiment: select 1 of these 250 people at random.

$N = 250$

Find $P(M) = \frac{N(M)}{N} = \frac{100}{250} = .4$

$P(H) = \frac{N(H)}{N} = \frac{90}{250} = .36$

} marginal probabilities

(2)

Find $P(M \cap H) = \frac{N(M \cap H)}{N} = \frac{30}{250} = .12$

This is a joint probability

Given that the person selected was male, find the probability that he has a high income

$P(H \mid M) = \frac{30}{100} = \frac{N(H \cap M)}{N(M)}$

"given"

(3)

<u>Defn</u>. The <u>conditional probability</u> of A, given B,

is $$P(A|B) = \frac{N(A \cap B)}{N(B)}$$

Note: $$P(A|B) = \frac{N(A \cap B)/N}{N(B)/N} = \frac{P(A \cap B)}{P(B)}$$

$$\therefore \boxed{P(A \cap B) = P(B)\,P(A|B)}$$

Multiplication rule for intersections

(or $P(A \cap B) = P(A)\,P(B|A)$)

(4)

<u>Defn</u>: If $P(A|B) = P(A)$, then

A & B are <u>independent</u>

Note: if A & B are independent, then

$$P(A \cap B) = P(A)\,P(B)$$

Check to see if "female" and "high income" are independent.

Compare $P(F|H)$ with $P(F)$

or $P(H|F)$ with $P(H)$

or $P(F \cap H)$ with $P(F)\,P(H)$

$$P(F|H) = \frac{60}{90} = 2/3$$
$$P(F) = \frac{150}{250} = \frac{3}{5}$$

Different, so F & H are dependent

(5)

Conditional probability example

Suppose that 1 out of 100 people are affected by a certain disease.

If the person has the disease, our test will correctly identify it 97% of the time.

(3% false negative rate)

(6)

If the person does not have the disease, the test will be correct 95% of the time

(5% false positive rate)

Question: If a person tests positive, what is the probability that they have the disease?

D: disease $\qquad P(D) = .01$

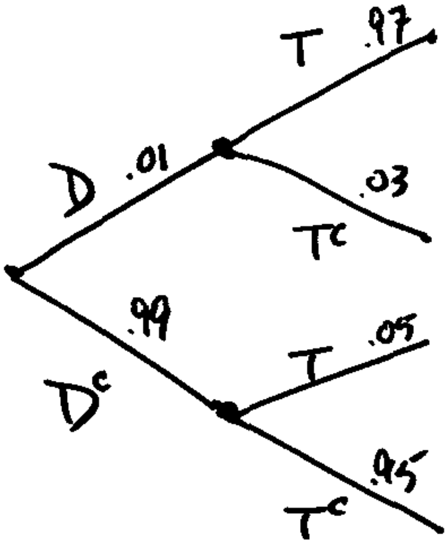
T: test is positive $\qquad P(T|D) = .97$

$P(T^c|D^c) = .95$

Question: $P(D|T)$

7

$P(D)\cdot P(T|D) = P(D\cap T)$



$(.01)(.97) = .0097$

$.0003 = P(D\cap T^c)$

$.0495 = P(D^c \cap T)$

$.9405 = P(D^c \cap T^c)$

8

true positives

false negatives

	T	T^c	
D	.0097	.0003	.01
D^c	.0495	.9405	.99
	.0592	.9408	1

true negatives

false positives

$$P(D|T) = \frac{P(D\cap T)}{P(T)} = \frac{.0097}{.0592} = .164$$

HW #2 due Thu 1/25

p.72 #7, 8, 9, 13

p.78 #4, 6

7. An information technology company will assign four electrical engineers to four different JAVA programming projects (one to each project). How many different assignments are there?

8. In many countries the license plates consist of a string of seven characters such that the first three are letters and the last four are numbers. If each such string of seven characters is equally likely, what is the probability that the string of three letters begins with a W and the string of four numbers begins with a 4? (*Hint.* Assume an alphabet of 26 letters. The number of possible such license plates is found in Example 2.3-4(c).)

9. Twelve individuals want to form a committee of four.

(a) How many committees are possible?

(b) The 12 individuals consist of 5 biologists, 4 chemists, and 3 physicists. How many committees consisting of 2 biologists, 1 chemist, and 1 physicist are possible?

(c) In the setting of part (b), if all committees are equally likely, what is the probability the committee formed will consist of 2 biologists, 1 chemist, and 1 physicist?

13. Five of the 15 school buses of a particular school district will be selected for thorough inspection. Suppose four of the buses have developed a slight defect since their last inspection (the steering wheel shakes when braking).

(a) How many possible selections are there?

(b) How many selections contain exactly three buses with the defect?

(c) If the five buses are selected by simple random sampling, what is the probability the sample includes exactly three of the buses with the defect?

(d) If the buses are selected by simple random sampling, what is the probability all five buses are free of the defect?

4. The PMF of the sum of two die rolls, found in Example 2.3-13, is

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

(a) For each of the following events specify the outcomes that belong to them, and use relation (2.4.1) to find their probabilities.

(i) $E_1 = \{$the sum of the two die rolls is at least 5$\}$.

(ii) $E_2 = \{$the sum of the two die rolls is no more than 8$\}$.

(iii) $E_3 = E_1 \cup E_2$, $E_4 = E_1 - E_2$, and $E_5 = E_1^c \cap E_2^c$.

(b) Recalculate the probability of E_3 using part (1) of Proposition 2.4-2.

(c) Recalculate the probability of E_5 using De Morgan's first law, the probability of E_3, and part (4) of Proposition 2.4-1.

6. Each of the machines A and B in an electronics fabrication plant produces a single batch of 50 electrical components per hour. Let E_1 denote the event that, in any given hour, machine A produces a batch with no defective components, and E_2 denote the corresponding event for machine B. The probabilities of E_1, E_2, and $E_1 \cap E_2$ are 0.95, 0.92, and 0.88, respectively. Express each of the following events as set operations on E_1 and E_2, and find their probabilities.

(a) In any given hour, only machine A produces a batch with no defects.

(b) In any given hour, only machine B produces a batch with no defects.

(c) In any given hour, exactly one machine produces a batch with no defects.

(d) In any given hour, at least one machine produces a batch with no defects.