Stat 451
12-1-16

(1)

Suggested practice problems for final:

p.323 #2c : C.I. for $\mu_1 - \mu_2$

p.323 #8b : C.I. for $p_1 - p_2$

p.343 #2c : C.I. for μ_d (matched pairs)

p.280 #2 : prediction interval

Final exam topics : - Central Limit Theorem
- Probabilities concerning $\bar{X}$
- Confidence intervals + sample size
- MOM & MLE

(2)

Probabilities about $\bar{X}$

Ex: Take a random sample of 15 observations from a population whose mean is $\mu = 30$ and st. dev. is $\sigma = 5$

1) Find the prob. of seeing an observation larger than 32. $P(X > 32)$ Not enough info.

2) Find the prob. that the average of the 15 obs. is larger than 32.

$$P(\bar{X} > 32)$$

③

Using the central limit theorem,

$$\bar{x} \sim N\left(30, \frac{5}{\sqrt{15}}\right)$$

$$\mu_{\bar{x}} = \mu, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$P(\bar{x} > 32) = P\left(Z > \frac{32-30}{5/\sqrt{15}}\right)$$

$$= P(Z > 1.55)$$

$$= .0606$$

④

Ex from HW

$$f(x) = \frac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}} \qquad x > 0$$

$$\mu = \theta\sqrt{\frac{\pi}{2}}$$

MOM: Set $\bar{x} = \mu$ & solve for θ

$$\bar{x} = \theta\sqrt{\frac{\pi}{2}} \qquad \hat{\theta}_{MOM} = \frac{\bar{x}}{\sqrt{\frac{\pi}{2}}}$$

MLE: $lik(\theta) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \frac{x_i}{\theta^2} e^{-\frac{x_i^2}{2\theta^2}}$

(5)

$$\mathcal{L}(\theta) = \ln lik(\theta) = \ln\left(\pi x_i \, \theta^{-2n} e^{-\frac{1}{2\theta^2}\sum x_i^2}\right)$$

$$= \ln(\pi x_i) - 2n\ln\theta - \frac{1}{2\theta^2}\sum x_i^2$$

$$\mathcal{L}'(\theta) = -\frac{2n}{\theta} - \frac{1}{2}\sum x_i^2(-2\theta^{-3}) \overset{set}{=} 0$$

$$-2n\theta^2 + \sum x_i^2 = 0$$

$$\theta^2 = \frac{\sum x_i^2}{2n}$$

$$\hat{\theta}_{MLE} = \sqrt{\frac{\sum x_i^2}{2n}}$$

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Verify that $\mu = \theta\sqrt{\frac{\pi}{2}}$

$$\mu = E[X] = \int_0^\infty x f(x)\,dx$$

$$= \int_0^\infty x \, \frac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}} \, dx$$

$$= \int_0^\infty \frac{x^2}{\theta^2} e^{-\frac{x^2}{2\theta^2}} \, dx$$

let $u = x$ $\quad dv = \frac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}} dx$

$du = dx$ $\quad v = -e^{-x^2/2\theta^2}$

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$$\mu = \cancel{-xe^{-x^2/2\theta^2}\Big|_0^\infty} + \int_0^\infty e^{-x^2/2\theta^2}\,dx$$

$$\frac{x}{e^{x^2/2\theta^2}} \rightarrow \frac{1}{\frac{x}{\theta^2} e^{x^2/2\theta^2}} \rightarrow 0$$

$$\mu = \int_0^\infty e^{-x^2/2\theta^2}\,dx$$

Let $z = \frac{x}{\theta}$ $\quad dz = \frac{dx}{\theta}$

$$= \int_0^\infty e^{-\frac{1}{2}z^2}\,\theta\,dz$$

Know:

$$\int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz = 1$$

$$\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz = .5$$

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$$\mu = \theta\sqrt{2\pi}\underbrace{\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz}_{.5}$$

$$= \theta\sqrt{2\pi}\cdot\frac{1}{2}$$

$$= \theta\sqrt{\frac{\pi}{2}}$$

For the exam: Tues 10:15-12:05

1 page of notes, Calculator, z, t, χ^2

Sample size for C.I. for μ

$$n = \left[\frac{Z_{\alpha/2}\, s}{E}\right]^2$$

desired margin of error

Sample size for C.I. for p

$$n = \frac{Z_{\alpha/2}^2\, \hat{p}\hat{q}}{E^2}$$

(Set $\hat{p} = \hat{q} = .5$ to get an overestimate of n)