Stat 451
11-29-16

(1)

Paired sample

Male	Female	$d = M - F$
9	10	-1
8	9	-1
6	6	0
7	9	-2
7	8	-1
5	7	-2
8	8	0

$\bar{d} = -1$
$s_d = .8165$
$n = 7$

Find a 95% conf. interval for the mean difference μ_d

Use the 1-sample t confidence interval $\left(\bar{x} \pm t \frac{s}{\sqrt{n}}\right)$

$$\bar{d} \pm t \frac{s_d}{\sqrt{n}}$$

.025 column
$df = n-1 = 6$

$$-1 \pm (2.447)\frac{.8165}{\sqrt{7}}$$

$\boxed{(-1.755, -.245)}$ OR $\boxed{-1 \pm .755}$

(2)

Prediction interval

Suppose we sample n items from a normal population with mean μ and std. dev. σ

Goal: predict the value of a new observation & give a 95% prediction interval

Use $\bar{X}$ as our best possible predictor of the new value.

Call the new value X.

Consider the random variable $X - \bar{X}$

(3)

Fact: $X - \bar{X}$ will again be normally distributed

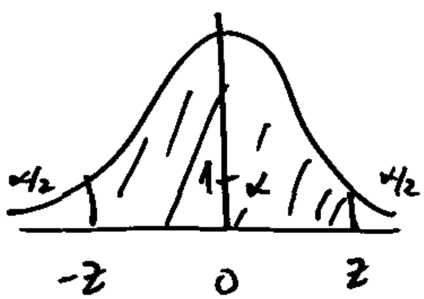
$$E[X-\bar{X}] = E[X] - E[\bar{X}]$$
$$= \mu - \mu = 0$$

$$V[X-\bar{X}] = V[X] + V[\bar{X}] + 0$$
$$= \sigma^2 + \frac{\sigma^2}{n} = \sigma^2\left(1 + \frac{1}{n}\right)$$

So $X - \bar{X} \sim N\left(0, \sigma^2\left(1+\frac{1}{n}\right)\right)$

$$\frac{X - \bar{X} - 0}{\sigma\sqrt{1+\frac{1}{n}}} \sim N(0,1)$$

(4)



$$P\left(-z < \frac{X-\bar{X}}{\sigma\sqrt{1+\frac{1}{n}}} < z\right) = .95$$

$$\bar{X} - z\sigma\sqrt{1+\frac{1}{n}} < X < \bar{X} + z\sigma\sqrt{1+\frac{1}{n}}$$

$\therefore$ $\bar{X} \pm z\sigma\sqrt{1+\frac{1}{n}}$ is a $1-\alpha$ prediction interval for the new value X.

(5)

$$\boxed{\bar{x} \pm t\, s\sqrt{1+\frac{1}{n}}} \text{ is a } 1-\alpha \text{ P.I. for } x$$

t: $\alpha/2$ column, $n-1$ df

Compare to $\bar{x} \pm t\, s/\sqrt{n}$ $\quad 1-\alpha$ C.I. for μ

Notice that the P.I. is wider than the C.I.

(6)

Assumptions needed for all of our confidence & prediction intervals

		Assume:
C.I. for μ:	$\bar{x} \pm t \frac{s}{\sqrt{n}}$	Either a normal population or n is large
C.I. for p:	$\hat{p} \pm z\sqrt{\frac{\hat{p}\hat{q}}{n}}$	Moderate n
C.I. for σ^2:	$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$ σ: $\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}$	Normal population

	Assume:
C.I. for $\mu_1 - \mu_2$: $\bar{x}_1 - \bar{x}_2 \pm t\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ df = long formula	Either the 1^{st} pop is normal or n_1 is large **AND** Either the 2^{nd} pop is normal or n_2 is large
C.I. for $p_1 - p_2$: $\hat{p}_1 - \hat{p}_2 \pm z\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$	Moderate n_1 & n_2

	Assume:
C.I. for μ_d : $\bar{d} \pm t\frac{s_d}{\sqrt{n}}$	Either a normal population of differences or large n
P.I. for x : $\bar{x} \pm t\, s\sqrt{1 + \frac{1}{n}}$	normal population