Stat 451
11-8-16
(1)

HW #5 due 11-15-16

p.165 #3

p.180 #17

p.191 #8

Example 1

Draw 1 slip of paper out of a container with 1,2,3,4

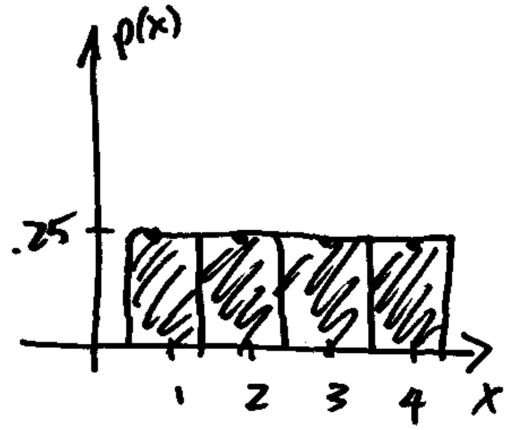
x	$p(x)$
1	.25
2	.25
3	.25
4	.25
	1

Example 2

Draw 2 slips of paper, with replacement & record $\bar{x}$

$\bar{x}$ (1st \ 2nd)	1	2	3	4
1	1	1.5	2	2.5
2	1.5	2	2.5	3
3	2	2.5	3	3.5
4	2.5	3	3.5	4

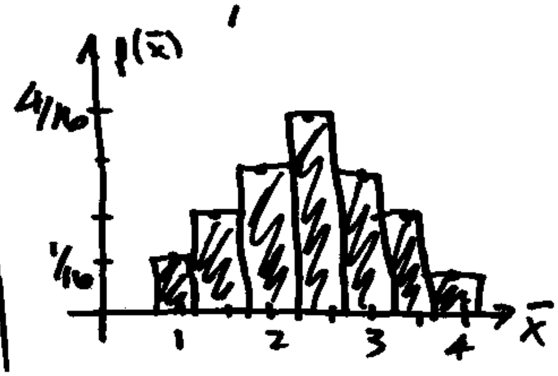
(2)



$$\mu_x = E[X] = \sum x\,p(x)$$
$$= 1(.25)+2(.25)+3(.25)+4(.25)$$
$$= 2.5$$

$$E[X^2] = 1^2(.25)+2^2(.25)+3^2(.25)+4^2(.25)$$
$$= \frac{1}{4}(1+4+9+16) = \frac{15}{2}$$

$\bar{x}$	$p(\bar{x})$
1	1/16
1.5	2/16
2	3/16
2.5	4/16
3	3/16
3.5	2/16
4	1/16
	1



$$\sigma_x^2 = E[X^2] - \mu^2$$
$$= \frac{15}{2} - \left(\frac{5}{2}\right)^2$$
$$= \frac{5}{4} = 1.25$$

(3)

$$\mu_{\bar{x}} = E[\bar{X}] =$$
$$1\left(\tfrac{1}{16}\right) + 1.5\left(\tfrac{2}{16}\right) + \ldots + 4\left(\tfrac{1}{16}\right)$$
$$= 2.5$$
$$\sigma_{\bar{x}}^2 = .625$$

(4)

Central Limit Theorem

If $X_1, \ldots, X_n$ are independent, identically distributed random variables, each having mean μ and variance σ^2, then as $n \to \infty$, the random variable $\dfrac{\bar{X} - \mu}{(\sigma/\sqrt{n})}$ has a distribution that tends to $N(0,1)$.

Consequence: For large values of n,

$\bar{X}$ is approximately normal with

$$\mu_{\bar{x}} = \mu \quad \text{and} \quad \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

(5)

Example: A certain electronic component has a lifetime whose mean is 2 years and whose std. dev. is .5 yr. We suspect that the distr. is slightly skewed to the right.

Find the prob. that a component lasts longer than 2.5 years.

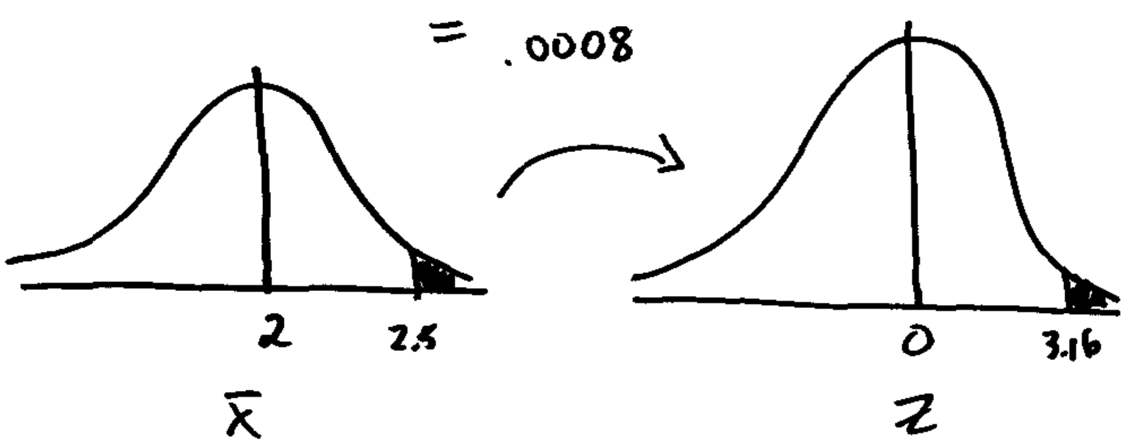
$P(X > 2.5)$ Not enough info.

(6)

Observe 10 components. Assume that their lifetimes are independent. Find the prob. that their average lifetime is more than 2.5 yrs.

$$P(\bar{X} > 2.5) \approx P\left(Z > \frac{2.5-2}{.5/\sqrt{10}}\right) = P(Z > 3.16)$$

$$= .0008$$



(7)

$$P\left(-1.96 < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < 1.96\right) \approx .95$$

$$-1.96 < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < 1.96$$ is an event whose prob. is $\approx .95$

$$-1.96 < \bar{x}-\mu < 1.96\frac{\sigma}{\sqrt{n}}$$

$$-\bar{x}-1.96 < -\mu < -\bar{x}+1.96\frac{\sigma}{\sqrt{n}}$$

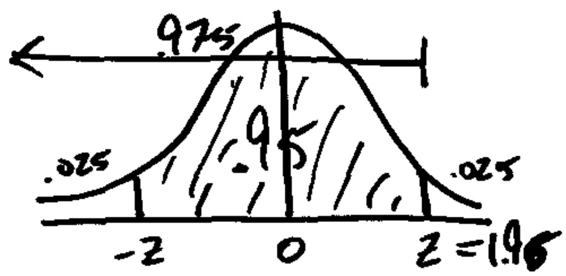
$$\bar{x}+1.96 > \mu > \bar{x}-1.96\frac{\sigma}{\sqrt{n}}$$

$$\boxed{\bar{x}-1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x}+1.96\,\sigma/\sqrt{n}}$$ is true with 95% confidence

(8)

Example: In a sample of 25 people, we find an average height of 70" and a standard deviation of 3". Estimate the average height in the population and give a margin of error of your estimate.

Central Limit Theorem said $\frac{\bar{x}-\mu}{(\sigma/\sqrt{n})}$ is approx $N(0,1)$



$$P(-1.96 < Z < 1.96) = .95$$

This says that:

if $\bar{x}$ is used as an estimate of μ,

it is off by at most $\underbrace{1.96\frac{\sigma}{\sqrt{n}}}$ (with 95% conf.)

margin of error

Summary: A 95% Conf. int. for μ is given by

$$\boxed{\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}}$$

Use s as an est. of σ, provided n is sufficiently large

In our example: $70 \pm 1.96\frac{3}{\sqrt{25}}$

$$\boxed{70 \pm 1.176}$$